Zonal Anisotropy: how to model the variogram?

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When the experimental variograms show different behaviors in different directions an anisotropic variogram model should be fitted to them. There are however some practical difficulties in fitting the most appropriate model. These difficulties can lead to a wrong model of the true underlying structure. When the directional variograms show different variability for the different directions the sills are not comparable. In such a case geostatistics can be used to establish a variogram model made up of components having a so-called "zonal" anisotropy.

Let's describe the procedure to be followed for calculating and fitting a variogram model with zonal anisotropy by making use of some bathymetric data, kindly provided by IFREMER (Institut Français de Recherche pour l'Exploitation de la Mer).

The data shown on figure 1 comes from a bathymetric survey originally conducted by the French hydrographic survey (SHOM) covering an area of approximately 8km x 5km along the Spanish and French mediterranean coast. Despite the non stationarity of the depth when moving away from the wast line, the sampling density authorizes to use variograms within a limited neighborhood. So we have to calculate and model directional variograms.

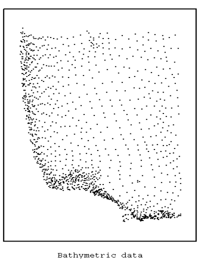

Figure 1: Base map of the bathymetric data

The first thing to do is to identify the 2 directions showing the minimum and maximum spatial continuity. The variogram map may help determine these directions.

Figure 2 shows the variograms calculated along N10W and N80E and the model that has been fitted. A variogram with a zonal component is required here. To simplify the fitting procedure, two directions have been fitted independently. That means that the resulting 2D model has the following form (where u and v represent the two main directions of anisotropy):

$$\gamma(h_u, h_v) = \gamma_1(h_u) + \gamma_2(h_v)$$

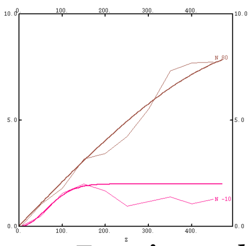

Figure 2: Experimental and modeled variogram in the 2 main directions of anisotropy

here γ(2D)=Gauss(along N10W,200m) +Spherical(along N80E,500m)

This model is "authorized" and fits well the two main directions, because the gaussian component has no effect along N80E and conversely the spherical component is not present along N10W. However this model is not satisfactory along intermediate directions, as shown in the left hand graphic of the figure 3.

The model along the bisecting direction N35E is greater than the model in N80E direction, which is supposed to be the direction of greatest variability!!!

The right approach to model all directions consistently with the observations made on the two main directions of variability will consist in fitting a model with two components: the first component is isotropic and the second zonal anisotropic.

The sill of the second component is equal to the difference between the sills observed along the main directions of continuity:

$$\gamma(h_u, h_v) = \gamma_1(\sqrt{h^2{}_u + h^2{}_v}) + \gamma_2(h_v)$$

In the first model the total sill is in fact the sum of the component sills in both directions. In the second model the total sill always lies between the sill of the isotropic component and the sill fitted to the variogram in the direction of maximum variability (v). To fit such a model that is correct for all directions some compromises must be made along the two main directions N80E and N10W. In our case the resulting fit (seen on the right hand graphic of Figure 3) is made up of:

γ(2D)= Gaussian (isotropic,170m) + Linear(along N80E)

While the consequences of the first model may be not so important when kriging from very dense data like in our bathymetric study, they may be disastrous when simulating or estimating 3D data.

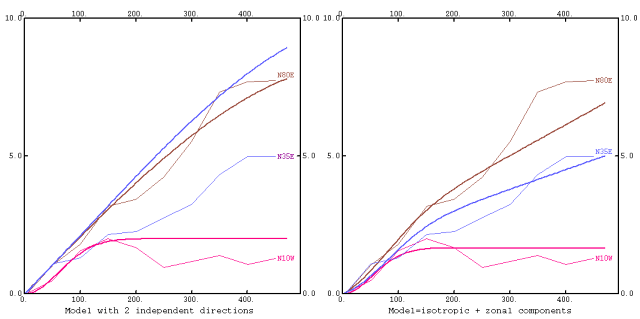

Figure 3: Variograms in the 2 main directions of anisotropy + a bisecting direction

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