Election Forecasts Using Spatiotemporal Models

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There exists a large literature on the problem of forecasting election results. But none of the published methods take spatial information into account, although there is clear evidence of geographic trends. To fill this gap, we use geostatistical procedures to build a spatial model of voting patterns. We test the model in three close elections and find that it outperforms rival methods in current use. We apply kriging (a spatial model) and cokriging (in a spatiotemporal model version) to improve the accuracy of election night forecasts. We compare the results with actual outcomes and also to predictions made using models that use only historical data from polling stations in previous elections. Despite the apparent volatility leading up to the three elections in our study, the use of spatial information strongly improves the accuracy of the prediction. Compared with forecasts using historical data alone, the spatiotemporal models are better whenever the proportion of counted votes in the election night tally exceeds 5%.

KEY WORDS: Cokriging; Geography; Kriging; Spatial correlation; Vote predictions.

1. INTRODUCTION

Numerous procedures have been suggested for forecasting final election outcomes. Following the basic classification proposed by Mughan (1987), all of these seem to be combinations (of varying degrees of complexity) of opinion polls, economic models, and incremental models. Opinion poll strategies use parties' standings in political surveys to predict their relative chances in the election (see Whiteley 1979; Bernardo 1984; Erikson and Sigelman 1995; Lewis-Beck and Tien 1999). Economic methods posit links between macroeconomic short-term fluctuations and patterns of party voting to explain and forecast election results. Direct illustration of such models may be found in Fair's works (e.g., Fair 1988, 1996), and more sophisticated examples are provided in the work of Jerôme, Jerôme, and Lewis-Beck (1999), who combined economic data and political issues, and Campbell and Wink (1990) and Brown and Chappell (1999), who also used pre-election polls. Incremental methods hold that the best signal for predicting a population's behavior is based on its previous behavior (Premfors 1981). They typically estimate current outcomes by combining past results with opinion poll data, election night declared results, or both. Incremental forecasts have been used by Rallings and Thrasher (1999), who predicted British party support from local by election results; Pavía-Miralles (2005), who predicted final election outcomes by comparing past and incoming polling station vote proportions; and Bernardo and Girón (1992), who combined campaign polls and early incoming results to sequentially predict outcomes. Brown, Firth, and Payne (1999) used opinion polls and economic and incremental ideas. They estimated the change in the share of the vote for each major party in the United Kingdom in each district using a regression model that combines actual election outcomes, party shares of the vote in the previous election, dummy variables, and several socioeconomic variables.

Despite the large literature, however, it does not appear that a model with spatial correlation has yet been tried [the so-called "spatial model of voting" does not use geography (see, e.g., Merill 1994)], although, similar to other social and economic variables, the geographical distribution of votes is not random. In this article we explore the use of information provided by the spatial structure of votes to forecast election outcomes. In particular, using techniques developed in geostatistics (cf. Cressie 1993; Chilès and Delfiner 1999; Wackernagel 2003), we predict the share of votes won by the main parties in the city of Valencia in the 1995 Corts Valencianes election and for the city of Madrid in the May 2003 and October 2003 Asamblea de Madrid elections. The estimates use polling stations as basic locations in the context of election night forecasting. The idea is to forecast the nonavailable polling stations (NAPS) from the current polling stations' incoming results and then, by aggregation, predict the final outcomes. Three alternative sets of predictions—spatial, spatiotemporal, and temporal forecasts—are obtained in each election for eight different moments in the election night. Spatial forecasts are based on kriging and provide estimates for each NAPS from the vote distribution observed in the stations in its vicinity. Spatiotemporal forecasts use cokriging to find NAPS estimates based on both the spatial distribution of the vote and the relationship that exists at each station between votes of consecutive elections. Finally, temporal forecasts are made to provide comparison with the spatial strategies, using the approach of Pavía-Miralles (2005), which exploits the consistency that polling stations show across elections.

The rest of the article is organized as follows. Section 2 investigates the relationship between location and electoral outcomes. Section 3 describes methodological issues. Section 4 compares the relative quality of the different forecasts, and Section 5 summarizes the findings.

2. SPACE AND ELECTIONS

Obviously, the support that political parties receive varies from place to place. Most previous research on this variation has looked for explanatory socioeconomic covariates to analyze why this happens. The main tools used by political scientists and geographers have been survey data of individual voters and aggregate data from small areas. Despite the complexity of the

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problem, given the number of interacting factors, researchers seem to have concluded that along with personal characteristics of residents, local factors also influence election outcomes (Clem 2006). Both individual background (e.g., class, partisan identification, religion, economic status, occupation, gender, education) and local context (e.g., neighbors, historical settlement patterns, community interests, diversity, local impact of economic issues, campaign visits, electoral system) interact to determine the political behavior of voters. Thus, some spatial autocorrelation patterns in electoral results is expected. The study of those patterns did not attract researchers until recently (e.g., Kohfeld and Sprague 1995; O'Loughlin 2002) with the widespread deployment of geographic information systems, despite the fact that the geographical structure of almost every election is evident in maps showing election outcomes.

Mapping election outcomes has become such a common tool among the media that, for example, the so-called "red and blue America" phenomenon is familiar to everyone and is used in popular discourse on U.S. elections. Thus, in the tradition of Cleveland (1993), to emphasize the prominence of geographical patterns in electoral results, we provide maps of three examples taken from elections in three countries. In contrast to Clem (2006), who underlined the importance of scale in interpreting the spatial patterns of outcomes, we use a different scale for each case.

Figure 1 shows a spatial representation, at the länders regional level, of the final outcomes for the 2005 German Bundestag election held on September 18, 2005. The Christian Democratic Union (CDU) reached plurality with 40.9% of the votes and 226 seats in the Bundestag (the German parliament). The Social Democratic Party of Germany (SPD), with 38.4% of the ballots and 222 seats, was close behind the CDU. Both parties, CDU and SPD, agreed to form a new government. Three additional groups also achieved representation in the Bundestag: the Free Democratic Party, the Left Party (LP), and the Alliance90–Greens coalition. All three groups together attained a total of 18.1% of votes and 166 seats. As can be seen in Figure 1, even at this coarse scale of regional aggregation, spatial patterns are apparent. Whereas the CDU won the majority in the southern länders (Rhineland-Palatinate, Baden-Württemberg, Bavaria, and Saxony), except for Saar, the SPD was the most successful party in the rest of the country, especially in the northwestern länders. In fact, it seems that SPD support decreases when moving from west to east. Also, although not shown in Figure 1, there is strong geographic dependence in the LP results. The LP was the third most popular party in the former East Germany, where it obtained 24.9% of votes, whereas in Western Germany, it obtained only 4.8%.

Figure 2 shows, at the county level, the outcomes in California for the 2004 U.S. presidential election. Almost all of the votes were divided between the main candidates: Republican George W. Bush and Democrat John F. Kerry. Kerry enjoyed victory in California with 54.31% of the votes, despite obtaining plurality in only 22 of 58 counties (Alameda, Alpine, Contra Costa, Humboldt, Imperial, Lake, Los Angeles, Marin, Mendocino, Mono, Monterey, Napa, Sacramento, San Benito, San Francisco, San Mateo, Santa Barbara, Santa Clara, Santa Cruz, Solano, Sonoma, and Yolo). Figure 2 shows that most of Kerry's counties are near the coast and around the San Francisco Bay area. Furthermore, except for five counties (Alpine, Imperial, Los Angeles, Mono, and Santa Barbara), Kerry's counties are contiguous.

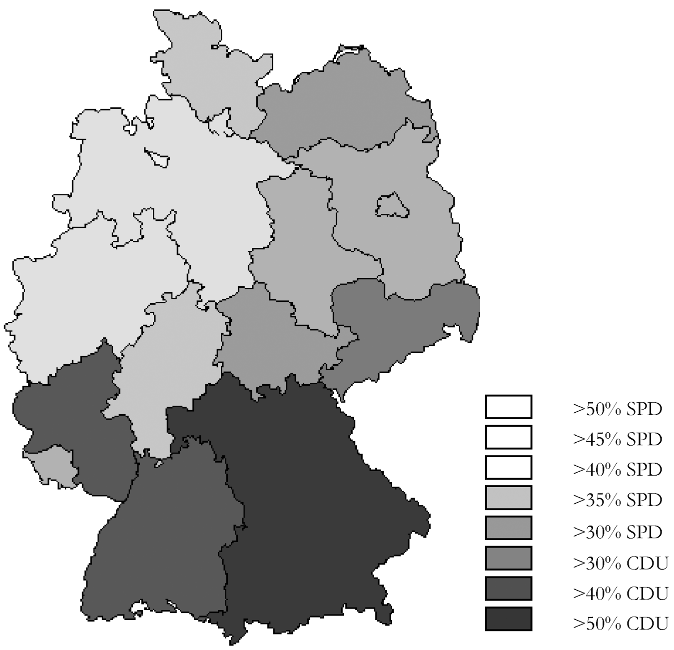

Figure 1. Mapping at the länders level of 2005 German Bundestag final election results. SPD, Social Democratic Party of Germany. CDU, Christian Democratic Union (in Bavaria, Christian Social Union). The party listed next to each box of the key indicates which party reached plurality for the corresponding color. (Data from www.electionresources.org.)

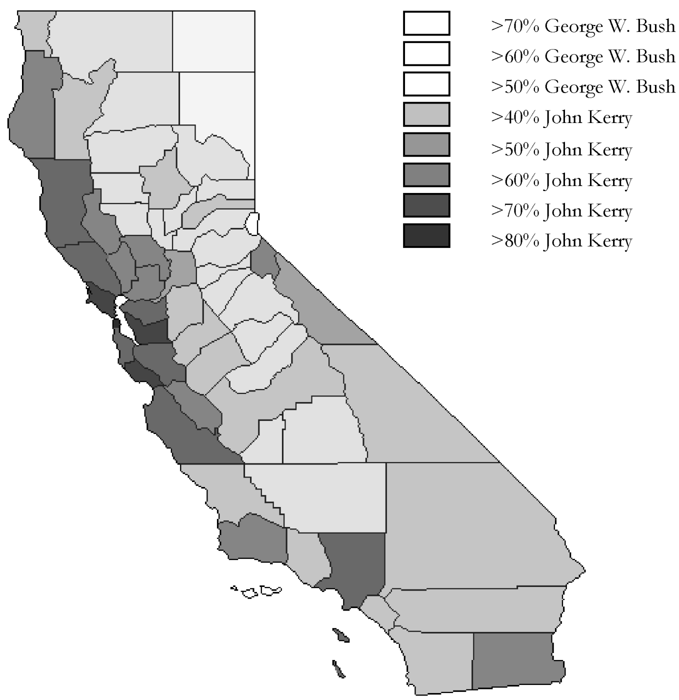

Figure 2. Mapping at the county level of the 2004 U.S. presidential election results in California. The name listed next to each box in the key indicates which candidate reached plurality for the corresponding color. (Data from *vote2004.ss.ca.gov*.)

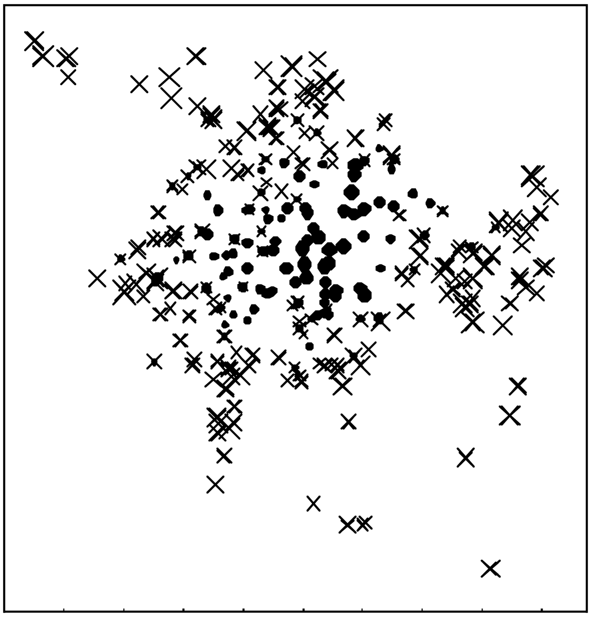

Figure 3. Election pluralities at the polling station level for the 1991 Corts Valencianes election in the city of Valencia, Spain. • denotes stations where the PP obtained plurality; × denotes stations where the PS reached plurality. The size of the symbols is proportional to the share of votes obtained by the corresponding party. (Data from Abacus 1991.)

Figure 3 shows, at the polling station level, the outcomes in the city of Valencia, Spain for the 1991 Corts Valencianes election held on May 26, 1991. In the whole Valencian region, the election winner was the socialist party (PS), with 40.92% of the votes, followed by the conservative party (PP) with 36.08%, the communist party (IU) with 12.40%, and the regional conservative party (UV) with 6.89%. In the city of Valencia, plurality was achieved by either the PS or the PP in 719 of its 730 polling stations. The UV achieved plurality in the 11 remaining stations (not shown in Fig. 3). The spatial structure of the outcomes is clear. Whereas PP had more followers in the city center, the PS had greater support in the suburbs. Furthermore, the sizes of the dots and ×'s suggest a progressive transition between PS-dominant and PP-dominant areas.

These three examples illustrate the pervasive importance of geography in elections. Interested readers can map other elections, using publicly available data at, for instance, *www.electionresources.org*, *www.uselectionatlas.org*, or *www.electoralgeography.com*.

3. THE FORECASTING METHODS

On election day, analysts use different approaches to predict the final results. The most common are exit polls, quick counts of a representative sample of polling stations, and forecasts based on provisional returns. Methods based on incoming results must deal with the fact that, especially in the early stages, the available data are not a random or representative sample of the election results. Several strategies have been proposed to deal with this issue (e.g., Bernardo and Girón 1992; Bernardo 1997; Brown et al. 1999; Pavía-Miralles 2005). These typically use a two-step procedure consisting of forecasting NAPS outcomes and then later combining forecasts and actual results to make an updated prediction. Of the different ways to handle NAPS, none exploits spatial structure to improve forecasts. In the present work, we obtain NAPS estimates through ordinary kriging (OK) and ordinary cokriging (OCK). This allows the use of models for spatial dependence.

Also, to calibrate the quality of the OK and OCK estimates, we compare their predictions those obtained from a procedure that uses only historical data (HD) (see Pavía-Miralles 2005). We chose this comparator because, unlike other nonspatial methods, it does not require external information (such as survey estimates or sociodemographic variables) and because it produces good estimates in elections for which the early returns converge slowly to the final results. The latter point is problematic for many forecasting methods (cf. the election night prediction of the 1995 Corts Valencianes in Bernardo 1997).

Suppose that N voters must choose among p competing parties, where the pth option consists of blank votes, null votes, and nonviable small parties. Let s be the number of polling stations among which the voters are divided, and let N_j be the number of voters at the jth polling station. Let x_{kj} be the proportion of votes that party k obtains in the current election at polling station j (location $\mathbf{b}_j$).

At a given time t on election day, only data from $s(t)$ polling stations [with $0 \le s(t) \le s$] are available. We assume that these correspond to the first $s(t)$ stations. Thus the problem involves obtaining estimates, $\hat{x}_{kj}$, for the $s - s(t)$ unobserved polling stations, and aggregating all of the available proportions (observed and forecast) to get a prediction of final outcomes. Estimates for the NAPS outcomes are found using the OK, OCK, and HD approaches.

Let z_k be the proportion of votes that party k receives from the population as a whole. Then a predictor of the final outcome distribution $\mathbf{z} = [z_1, z_2, \ldots, z_p]'$ is obtained (see the App.) by

$$\hat{\mathbf{z}} = \frac{\sum_{j=1}^{s(t)} \pi_j \omega_j \mathbf{x}_j + \sum_{j=s(t)+1}^{s} \hat{\pi}_j \omega_j \hat{\mathbf{x}}_j}{\sum_{j=1}^{s(t)} \pi_j \omega_j + \sum_{j=s(t)+1}^{s} \hat{\pi}_j \omega_j}, \tag{1}$$

where $\omega_j = N_j/N$ denotes the relative number of voters of polling station j, $\pi_j = n_j/N_j$ is the participation rate in polling station j, and $\mathbf{x}_j = [x_{1j}, x_{2j}, \ldots, x_{pj}]'$ is the p-dimensional column convex vector of proportions of votes that parties are receiving in station j, with $\hat{\pi}_j$ an approximation of π_j obtained by OK, OCK, or OLS.

3.1 Kriging and Cokriging Procedures

Both kriging and cokriging are based on the idea that the data are the realization of a stochastic process, X, over space. This approach applies to a wide range of phenomena (cf. Tzeng, Huang, and Cressie 2005; Spence et al. 2007) and implies dealing with an infinite family of random variables $X(\mathbf{s})$ constructed at all points $\mathbf{s}$ in a region. The variables take different values depending on the location and the correlation structure, and each observed datum $x(\mathbf{s})$ is supposed to be a realization of the process $X(\mathbf{s})$.

Observing the set of polling stations as a group of points in a map, the proportion of votes that each political option obtains could be considered a spatial process. At time t, outcomes have been reported only for locations $\mathbf{b}_1, \mathbf{b}_2, \ldots, \mathbf{b}_{s(t)}$, and so interpolation is used to predict the vote counts for the remaining stations. Geostatistics uses kriging and cokriging to take spatial dependence into account when interpolating. Kriging is a univariate procedure, and cokriging handles multivariate observations (e.g., proportions for multiple political parties). Kriging interpolates the values of the target variable at unobserved locations using observations of the same variable exclusively, whereas cokriging estimates the unknown values using additional information from covariates and estimates of the correlation structure among the multivariate data.

Moreover, kriging and cokriging—which are minimum mean squared error methods of spatial prediction—produce the best linear unbiased estimators in univariate and multivariate applications and use the covariance or variogram function (the spatial equivalent of the autocorrelation function in time series analysis) to account for the correlation structure when making interpolative predictions.

In our case, the process of interest is the number or proportion of votes cast for each of the p political parties at the different polling stations. Here the X_k—the proportion of votes that party k is collecting in the current election—is the main process; the random variables are $X_k(\mathbf{b}_j)$, the proportion of votes cast for the kth party at polling site $\mathbf{b}_j$; and the data are x_{kj}, the observed proportion for party k at the jth site. The proportions for other parties, $X_{k'}$, with $k' \neq k$, and the proportions in previous elections, Y_k, with $k = 1, \dots, p$, may be auxiliary processes of process X_k.

In the kriging estimate for party k at time t, the values $X_k(\mathbf{b}_j) = x_{kj}$, for $j = 1, 2, \dots, s(t)$ are available, and the predicted proportion of votes for that party at polling station j, $j \in \{s(t)+1, \dots, s\}$, is estimated as a weighted average of the proportion of votes obtained for that party at the stations that have already been counted through

$$\hat{x}_{kj} = \hat{X}_k(\mathbf{b}_j) = \sum_{m=1}^{s(t)} \lambda_m X_k(\mathbf{b}_m) = \sum_{m=1}^{s(t)} \lambda_m x_{km}. \tag{2}$$

The value of the estimate depends on the weights λ_m that are used.

Depending on the kind of stochastic process used, three different types of kriging can be distinguished: simple kriging, ordinary kriging (OK), and universal kriging. In our analysis, for the labeled spatial forecast strategy, we use OK to obtain the estimates of the NAPS proportions. This assumes that the random processes are intrinsically stationary [i.e., for every vector $\mathbf{d}$ linking any two locations $\mathbf{b}_j$ and $\mathbf{b}_j + \mathbf{d}$ in the map, $X(\mathbf{b}_j + \mathbf{d}) - X(\mathbf{b}_j)$ is a second-order stationary random process] with unknown means. Thus, requiring the classical conditions of unbiasedness, $E[\hat{X}_k(\mathbf{b}_j) - X_k(\mathbf{b}_j)] = 0 \Leftrightarrow \sum_{m=1}^{s(t)} \lambda_m = 1$, and minimum error variance $\min V[\hat{X}_k(\mathbf{b}_j) - X_k(\mathbf{b}_j)] = \min(2\sum_{i=1}^{s(t)} \lambda_i \gamma(\mathbf{b}_i - \mathbf{b}_j) - \sum_{i=1}^{s(t)}\sum_{m=1}^{s(t)} \lambda_i \lambda_m \gamma(\mathbf{b}_i - \mathbf{b}_m))$, we find, following for instance Montero and Larraz (2006, pp. 207–209), that the weights of (2) satisfy $\boldsymbol{\lambda} = \boldsymbol{\Gamma}^{-1}\boldsymbol{\Gamma}_0$, where

$$\boldsymbol{\Gamma} = \begin{pmatrix} \gamma(\mathbf{0}) & \gamma(\mathbf{b}_1 - \mathbf{b}_2) & \cdots & \gamma(\mathbf{b}_1 - \mathbf{b}_{s(t)}) & 1 \\ \gamma(\mathbf{b}_2 - \mathbf{b}_1) & \gamma(\mathbf{0}) & \cdots & \gamma(\mathbf{b}_2 - \mathbf{b}_{s(t)}) & 1 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \gamma(\mathbf{b}_{s(t)} - \mathbf{b}_1) & \gamma(\mathbf{b}_{s(t)} - \mathbf{b}_2) & \cdots & \gamma(\mathbf{0}) & 1 \\ 1 & 1 & \cdots & 1 & 0 \end{pmatrix},$$

$$\boldsymbol{\lambda} = \begin{pmatrix} \lambda_1 \\ \lambda_2 \\ \vdots \\ \lambda_{s(t)} \\ \alpha \end{pmatrix}, \qquad \boldsymbol{\Gamma}_0 = \begin{pmatrix} \gamma(\mathbf{b}_1 - \mathbf{b}_j) \\ \gamma(\mathbf{b}_2 - \mathbf{b}_j) \\ \vdots \\ \gamma(\mathbf{b}_{s(t)} - \mathbf{b}_j) \\ 1 \end{pmatrix},$$

$\mathbf{b}_i - \mathbf{b}_j$ is the vector that links (usually it is the distance between) polling stations i and j, α is a Lagrange multiplier, and $\gamma(\mathbf{d})$ is the variogram function that shows how the dissimilarity between pairs of observations evolves with separation $\mathbf{d}$, that is, for any pair of locations $\mathbf{b}_j$ and $\mathbf{b}_j + \mathbf{d}$, such that $\gamma(\mathbf{d}) = \frac{1}{2}V[X(\mathbf{b}_j + \mathbf{d}) - X(\mathbf{b}_j)]$.

In the cokriging analyses, we evaluated several sets of auxiliary processes to improve prediction. On balance, we believe that the best compromise between accuracy and simplicity is achieved by using, for each party k, a single auxiliary process: the process based on the proportions won by the same party in the previous election, Y_k. Thus at time t, the values $X_k(\mathbf{b}_j) = x_{kj}$ are available for $j = 1, 2, \dots, s(t)$, and also the results obtained by that party in the previous election in all the stations, $Y_k(\mathbf{b}_j) = y_{kj}$, for $j = 1, 2, \dots, s$ (y_{kj} is the proportion of votes achieved in the previous election by party k in polling station j, location $\mathbf{b}_j$). Thus some variables share sample locations; X_k is observed in $s(t)$ locations, and Y_k is observed in all s locations (a situation called *partial heterotopy*). Thus, under the hypothesis that X_k and Y_k are intrinsically stationary, the OCK estimator of X_k (the spatiotemporal forecast estimate for the NAPS) at the jth polling station is given by

$$\begin{aligned} \hat{x}_{kj} = \hat{X}_k(\mathbf{b}_j) &= \sum_{m=1}^{s(t)} \lambda_{xm} X_k(\mathbf{b}_m) + \sum_{m=1}^{s} \lambda_{ym} Y_k(\mathbf{b}_m) \\ &= \sum_{m=1}^{s(t)} \lambda_{xm} x_{km} + \sum_{m=1}^{s} \lambda_{ym} y_{km}. \end{aligned} \tag{3}$$

In the same way as in OK approach, the weights λ_{xm} and λ_{ym} are calculated to ensure that the estimator is optimal, in the sense that it is unbiased and has minimum error variance. Thus, following Wackernagel (2003, p. 161), the weights of (3) are found by solving the following $s + s(t) + 2$ equations:

$$\begin{cases} \displaystyle\sum_{m=1}^{s(t)} \lambda_{xm}\gamma_{xx}(\mathbf{b}_i - \mathbf{b}_m) + \sum_{m=1}^{s} \lambda_{ym}\gamma_{xy}(\mathbf{b}_i - \mathbf{b}_m) + \omega_x \\ \quad = \gamma_{xx}(\mathbf{b}_i - \mathbf{b}_j) \quad \text{with } i = 1, \dots, s(t) \\ \displaystyle\sum_{m=1}^{s(t)} \lambda_{xm}\gamma_{xy}(\mathbf{b}_i - \mathbf{b}_m) + \sum_{m=1}^{s} \lambda_{ym}\gamma_{yy}(\mathbf{b}_i - \mathbf{b}_m) + \omega_y \\ \quad = \gamma_{xy}(\mathbf{b}_i - \mathbf{b}_j) \quad \text{with } i = 1, \dots, s \\ \displaystyle\sum_{m=1}^{s(t)} \lambda_{xm} = 1, \qquad \sum_{m=1}^{s} \lambda_{ym} = 0, \end{cases} \tag{4}$$

where γ_{xx} is the direct variogram function of the process X_k, γ_{yy} is the direct variogram of Y_k, γ_{xy} is the cross-variogram between X_k and Y_k, and ω_x and ω_y are Lagrange multipliers. A brief description of the cokriging procedure has been given by Gotway and Young (2002), and details have been provided by Wackernagel (2003, pp. 159–161). Note that OCK reduces to OK when spatial estimation is carried out without auxiliary random processes.

The polling station participation rates are also estimated by applying kriging and cokriging, with Π, the current participation rate, as the main process [where $\Pi(\mathbf{b}_j) = \pi_j$] and Π_{-1}, the previous participation rate, as the auxiliary process, where $\Pi_{-1}(\mathbf{b}_j) = {}_{-1}\pi_j$ corresponds to the participation rate registered in station j in the previous election.

Cross and direct variograms are obtained through two-step procedure. First, using the classical variogram estimator based on the method of moments (Lark and Papritz 2003), rough point estimates of the variograms are found. Second, to ensure a positive definite model, a theoretical variogram function (Emery 2000, pp. 93–104 for the usual variogram models) has been fitted to the sequence of average dissimilarities, in keeping with the linear model of coregionalization (cf. Goovaerts 1997, pp. 108–115). The ISATIS v4.1.1 (2001) software is used to find the OK and OCK estimates.

3.2 HD Procedure

Based on the strong consistency that polling stations show between elections and on the fact that swings between parties spread across space, Pavía-Miralles (2005) suggested using past results from polling stations to predict NAPS outcomes. In particular, the HD strategy proposes that at the polling station level and for each party, the current and past election proportions of votes are linearly related, suggesting the usual linear regression model with constant correlations between parties and conditional independence between polling stations. That is, the HD model implies

$$x_{kj} = \alpha_k + \beta_k y_{kj} + e_{kj} \quad \text{for } k = 1, \ldots, p \text{ and } j = 1, \ldots, s, \tag{5}$$

where the e_{kj} are normal disturbances, with $E(e_{kj}) = 0$ and $E(e_{kj} e_{k*j*}) = \delta_{jj*}\sigma_{kk*}$, with δ the Kronecker delta function.

At time t, results from the first $s(t)$ stations are available. Thus let $\mathbf{y}_k = [y_{k1}, y_{k2}, \ldots, y_{ks(t)}]'$ and $\boldsymbol{\chi}_k = [x_{k1}, x_{k2}, \ldots, x_{ks(t)}]'$ be the $s(t) \times 1$ vectors of past and current proportions of votes obtained for party k in counted stations, and let $\mathbf{e}_k = [e_{k1}, e_{k2}, \ldots, e_{ks(t)}]'$ be a $s(t) \times 1$ vector of disturbances. Then, defining $\boldsymbol{\chi} = [\boldsymbol{\chi}'_1, \boldsymbol{\chi}'_2, \ldots, \boldsymbol{\chi}'_p]'$ as the $s(t) \cdot p \times 1$ vector of current proportions, $\boldsymbol{\beta} = [\alpha_1, \beta_1, \alpha_2, \beta_2, \ldots, \alpha_p, \beta_p]'$, as the $2p \times 1$ coefficient vector, and $\Upsilon = \text{block-diag}[\Upsilon_1, \Upsilon_2, \ldots, \Upsilon_p]$ as the $s(t) \cdot p \times 2p$ block-diagonal matrix of past proportions, with generic k block the $s(t) \times 2$ matrix $\Upsilon_k = [\mathbf{i}_{s(t)}, \mathbf{y}_k]$, for $\mathbf{i}_{s(t)}$ a $s(t) \times 1$ vector of 1's, the following linear relationship emerges:

$$\boldsymbol{\chi} = \Upsilon\boldsymbol{\beta} + \mathbf{e}, \tag{6}$$

where $\mathbf{e} = [\mathbf{e}'_1, \mathbf{e}'_2, \ldots, \mathbf{e}'_p]'$ is a $s(t) \cdot p \times 1$ normal mean-0 random vector of disturbances with covariance matrix $\boldsymbol{\Omega} = E(\mathbf{e}\mathbf{e}') = \mathbf{C} \otimes \mathbf{I}_{s(t)}$, $\mathbf{C}$ is the $p \times p$ matrix with generic $(k, k*)$-element σ_{kk*}, $\mathbf{I}_{s(t)}$ is the identity matrix of order $s(t)$, and $\otimes$ represents the Kronecker product.

Equation (6) represents, in observed values, a compact expression of (5) with singular and unknown covariance matrix $\boldsymbol{\Omega}$. Thus, following the iterative algorithm proposed by Pavía-Miralles (2005, p. 1121) and letting $\boldsymbol{\Omega}^+$ denote the Moore–Penrose generalized inverse of $\boldsymbol{\Omega}$, the BLUE of the parameters is

$$\hat{\boldsymbol{\beta}} = (\Upsilon'\boldsymbol{\Omega}^+\Upsilon)^{-1}\Upsilon'\boldsymbol{\Omega}^+\boldsymbol{\chi}. \tag{7}$$

The NAPS proportions, for the temporal forecast strategy, are found from (5) by using the parameter estimates of (7). In addition, estimates of the unavailable π_j are made by regressing current proportion participation rates on past rates (see app. B in Pavía-Miralles and Larraz-Iribas 2008).

4. FORECASTING ELECTION RESULTS

We have used the spatial structure of votes to forecast election outcomes by estimating the proportion of votes reached by the main parties in the city of Valencia in the 1995 Corts Valencianes election and in the city of Madrid in the May 2003 and in the October 2003 Asamblea de Madrid elections. These polls have been selected in order to test the predictive capacity of spatial strategies in difficult situations, where traditional election night forecasting methods had to cope with significant lags in the convergence of the incoming observations to the final outcomes.

Eight different time points were selected to assess the forecasting power of the techniques. Tables 1–3 present, for 1%, 2%, 2.5%, 5%, 7.5%, 15%, and 25% of polled votes, the forecasts that would have been obtained had OK (spatial forecasts), HD (temporal forecasts), and OCK (spatiotemporal forecasts) procedures been used to predict these three elections. The tables also present the number of polling stations available at each time point, the proportion of registered votes, and the predicted participation rates. Moreover, the error statistic, which measures the sum of the differences in absolute values between final outcomes and point forecasts, $\sum_k |z_k - \hat{z}_k|$, is shown.

4.1 1995 Corts Valencianes Elections

On May 28, 1995 the citizens of the Valencia region were called to elect the fourth regional parliament in their history. In the previous regional election, PS had obtained a majority of seats. For this election, only four parties had a real chance of winning parliamentary representation: PP, PS, IU, and UV. The issue was which combination of parties, either PP + UV or PS + IU, would reach a majority. The election night process of the 1995 Corts Valencianes vote was characterized by a great delay in the convergence of the counted votes to the final results. Thus, although PP + UV finally got 47 seats and PS + IU got 42 seats, it was necessary to poll nearly three-quarters of the votes to find a difference greater than one seat between the two party blocks. In the city of Valencia, the convergence lag was even more pronounced (e.g., when 25% of the votes were reported, the absolute aggregated errors were 3.47 for the region and of 8.80 for the city), and swings from the 1991 election were higher, even spectacular (see Table 1). These factors made real-time prediction difficult.

Table 1 provides the estimates that would have been obtained had kriging, HD, and cokriging methods been used on the 1995 Corts Valencianes election night in the city of Valencia (619,868 voters in 822 stations). Despite the environment of drastic political change and the lag in vote convergence, the spatial methods show great accuracy, even when the proportion of polled votes is small. Both spatial and spatiotemporal forecasts substantially improve provisional results. But spatial forecasts are systematically worse than the spatiotemporal predictions. Moreover, the spatial estimates seem to be more

Table 1. Forecasts of the 1995 Corts Valencianes election final results for the city of Valencia

	Polled[b]	Stations[c]	Participation[d]	Percentages[a]				
				PS	PP	IU	UV	Error[e]
1991 Results	100.00	730	63.83	36.03	27.08	8.21	19.24	
1995 Results	**100.00**	**822**	**73.85**	**25.36**	**46.59**	**14.09**	**10.38**	
Provisional results	**1.01**	10	71.45	32.46	37.53	15.41	10.93	18.01
Spatial forecasts			72.11	30.10	40.72	14.83	10.77	11.74
Temporal forecasts			72.94	26.07	43.88	15.06	11.36	5.37
Spatiotemporal forecasts			73.54	25.83	43.32	14.71	12.63	6.60
Provisional results	**2.09**	20	72.42	30.43	40.99	14.87	10.46	11.53
Spatial forecasts			73.01	28.01	43.78	14.56	10.42	5.96
Temporal forecasts			73.22	25.83	45.00	14.70	10.88	3.16
Spatiotemporal forecasts			73.67	25.95	44.74	14.70	10.99	3.65
Provisional results	**2.49**	24	72.37	30.03	41.60	14.65	10.51	10.34
Spatial forecasts			72.98	27.88	44.35	14.34	10.66	5.28
Temporal forecasts			73.29	25.82	45.32	14.49	11.10	2.84
Spatiotemporal forecasts			73.64	25.92	45.05	14.41	11.17	3.20
Provisional results	**5.04**	47	71.73	29.74	42.28	14.56	10.18	9.36
Spatial forecasts			72.79	26.88	46.01	14.03	9.91	2.63
Temporal forecasts			72.76	26.35	45.72	14.15	10.47	2.01
Spatiotemporal forecasts			73.22	26.03	45.45	14.07	10.99	2.45
Provisional results	**7.50**	69	71.53	30.01	42.32	14.27	10.14	9.34
Spatial forecasts			72.39	27.49	45.61	13.70	9.99	3.88
Temporal forecasts			72.69	26.41	45.61	14.05	10.63	2.33
Spatiotemporal forecasts			73.19	26.41	45.88	13.82	10.45	2.10
Provisional results	**9.98**	91	71.33	30.20	41.21	14.58	10.62	10.94
Spatial forecasts			72.46	27.43	45.39	13.78	10.26	3.70
Temporal forecasts			72.59	25.77	45.31	14.12	10.88	2.22
Spatiotemporal forecasts			73.15	26.20	45.79	14.04	10.72	2.03
Provisional results	**14.97**	133	71.72	29.77	41.09	14.80	10.96	11.20
Spatial forecasts			72.74	27.01	45.01	14.06	10.55	3.42
Temporal forecasts			72.88	25.78	45.36	14.24	11.05	2.45
Spatiotemporal forecasts			73.43	25.89	45.49	14.11	10.84	2.11
Provisional results	**25.02**	220	71.96	28.84	42.23	14.78	10.66	8.80
Spatial forecasts			73.26	26.38	45.90	13.83	10.45	2.05
Temporal forecasts			73.00	25.63	45.35	14.22	11.19	2.45
Spatiotemporal forecasts			73.58	25.82	45.87	14.10	10.62	1.42

NOTE: Calculated using data from Abacus (1991, 1995).

[a] The result lines show the proportions of valid votes that each political party was receiving at that time point of interest. The forecast lines portray the final vote's proportion forecasts obtained after applying the corresponding strategy.

[b] The percentage of census polled at each time point of interest appears in the result rows of this column.

[c] The number of stations polled at each time point appears in the result rows of this column.

[d] The result lines show the voters' participation rate at that time point. The forecast lines portray the final participation rate forecasts obtained after applying the corresponding strategy.

[e] The error column displays the sum of the differences in absolute values between final outcomes and provisional data (either results or predictions), $\sum_j |z_j - \hat{z}_j|$.

sensitive to transitory interim results and to require data from more locations to significantly reduce their level of error. Compared with HD estimates, both temporal and spatiotemporal forecasts are found to produce comparable error scores. Nevertheless, temporal forecasts appear to be better when very few polling stations have reported, and the spatiotemporal predictions dominate when the number of reporting stations reaches some critical threshold. Even purely spatial forecasts are better than purely temporal forecasts when the number of reporting stations is sufficiently large.

4.2 May 2003 Asamblea de Madrid Elections

On May 25, 2003, the voters of Madrid were called to renew their regional parliament. The changes introduced since the previous election (the incumbent President did not run, and the number of seats was increased from 101 to 111) led pundits to predict a hotly disputed race. Could PS, with the help of IU regain power after 8 years, or would PP renew its last two majorities with a new leader? The early returns presaged an easy PS victory, but as the count advanced, the differences between PS + IU and PP narrowed. In fact, over the whole region, PS +

Table 2. Forecasts of the May 2003 Asamblea de Madrid election final results for the city of Madrid

	Polled[b]	Stations[c]	Participation[d]	Percentages[a]			
				PS	PP	IU	Error[e]
1999 Results	100.00	3,047	62.37	33.93	54.09	6.99	
May 2003 Results	**100.00**	**3,217**	**72.75**	**37.90**	**48.66**	**7.21**	
Provisional results	**1.00**	31	72.02	41.90	45.35	7.51	7.61
Spatial forecasts			73.33	38.60	47.34	7.26	2.07
Temporal forecasts			72.20	37.89	47.97	7.05	.86
Spatiotemporal forecasts			72.32	37.92	48.08	6.81	1.01
Provisional results	**2.00**	68	71.99	44.64	41.44	8.17	14.93
Spatial forecasts			73.16	38.28	46.93	7.19	2.15
Temporal forecasts			72.50	38.09	47.37	7.45	1.72
Spatiotemporal forecasts			72.49	37.95	47.52	7.27	1.25
Provisional results	**2.51**	83	71.65	43.36	43.08	7.93	11.77
Spatial forecasts			72.65	38.27	46.93	7.19	2.13
Temporal forecasts			72.46	37.88	47.85	7.33	.95
Spatiotemporal forecasts			72.38	37.90	47.90	7.15	.82
Provisional results	**5.02**	165	71.56	42.98	43.22	7.75	11.06
Spatial forecasts			72.82	38.61	47.34	7.25	2.07
Temporal forecasts			72.62	38.34	47.41	7.25	1.73
Spatiotemporal forecasts			72.68	38.21	47.63	7.05	1.50
Provisional results	**7.51**	246	71.57	43.38	42.84	7.87	11.96
Spatial forecasts			72.93	38.52	47.43	7.24	1.89
Temporal forecasts			72.63	38.45	47.43	7.23	1.81
Spatiotemporal forecasts			73.15	38.28	47.70	7.08	1.49
Provisional results	**10.00**	325	71.12	44.08	42.06	7.98	13.56
Spatial forecasts			72.41	38.55	47.40	7.25	1.95
Temporal forecasts			72.32	38.44	47.46	7.33	1.86
Spatiotemporal forecasts			72.89	38.24	47.76	7.23	1.26
Provisional results	**14.99**	485	71.16	43.59	42.57	7.97	12.54
Spatial forecasts			72.36	38.31	47.63	7.24	1.48
Temporal forecasts			72.34	38.28	47.63	7.30	1.50
Spatiotemporal forecasts			72.35	38.06	47.83	7.20	1.01
Provisional results	**25.00**	806	71.10	43.56	42.42	8.11	12.81
Spatial forecasts			72.30	37.98	47.57	7.18	1.21
Temporal forecasts			72.38	38.23	47.40	7.26	1.64
Spatiotemporal forecasts			72.35	38.04	47.84	7.20	.98

NOTE: Calculated using data from ICM (1999, 2003a).

[a] The result lines show the proportions of valid votes that each political party was receiving at the time point of interest. The forecast lines portray the final votes proportion forecasts obtained after applying the corresponding strategy.

[b] The percentage of census polled at each time point of interest appears in the result rows of this column.

[c] The number of stations polled at each time point appears in the result rows of this column.

[d] The result lines show the voters' participation rate at that time point. The forecast lines portray the final participation rate forecasts obtained after applying the corresponding strategy.

[e] The error column displays the sum of the differences in absolute values between final outcomes and provisional data (either results or predictions), $\sum_j |z_j - \hat{z}_j|$.

IU only got 1% more ballots than PP and won the election by a single seat. In the city of Madrid, though PP won the election, its loss of support from the 1999 election was even greater than in the whole region (a 9.62-point loss compared with a 8.13-point loss).

Table 2 shows the forecasts that would have been attained had the OK, HD, and OCK strategies been used in the May 2003 Asamblea de Madrid election in the city of Madrid (2,348,226 voters divided into 3,217 stations). When analyzing predictions and errors, similar conclusions to those reached in the Valencian example emerge. In this case, this highlights that the errors are systematically smaller, spatial forecasts are quite good from the very beginning, and the superiority of spatiotemporal forecasts against temporal estimates occurs more quickly, with only 2% of votes polled.

4.3 October 2003 Asamblea de Madrid Elections

In October 2003, the citizens of Madrid region were called to elect their regional parliament for the second time in the year. Only a few months earlier, the results suggested a coalition between PS and IU; however, two representatives of PS were opposed to an accord with IU and boycotted the creation of any

Table 3. Forecasts of October-2003 Asamblea de Madrid election final results for the city of Madrid

	Polled[b]	Stations[c]	Participation[d]	Percentages[a]			
				PS	PP	IU	Error[e]
May 2003 Results	100.00	3,217	72.75	37.90	48.66	7.21	
October 2003 Results	**100.00**	**3,190**	**66.23**	**36.43**	**50.97**	**8.33**	
Provisional results	**1.00**	31	64.11	45.25	42.32	8.64	17.16
Spatial forecasts			64.32	40.91	46.62	8.68	8.48
Temporal forecasts			66.53	37.66	50.38	7.95	1.44
Spatiotemporal forecasts			66.28	37.92	50.11	8.13	2.15
Provisional results	**2.01**	63	63.36	45.49	41.47	8.91	17.98
Spatial forecasts			64.30	40.29	46.81	8.80	7.55
Temporal forecasts			65.78	37.14	50.41	8.10	1.04
Spatiotemporal forecasts			65.67	37.41	50.15	8.29	1.76
Provisional results	**2.51**	79	63.59	44.68	42.06	9.07	16.42
Spatial forecasts			64.24	40.06	47.05	8.87	7.01
Temporal forecasts			65.66	36.98	50.39	8.26	1.06
Spatiotemporal forecasts			65.62	36.99	50.32	8.38	1.16
Provisional results	**5.00**	155	63.16	45.84	40.69	9.38	18.64
Spatial forecasts			63.70	40.01	46.98	8.88	7.02
Temporal forecasts			65.61	36.83	50.41	8.47	.82
Spatiotemporal forecasts			65.52	36.99	50.40	8.37	1.09
Provisional results	**7.51**	231	63.01	45.82	40.95	9.19	18.55
Spatial forecasts			63.64	39.27	47.87	8.80	5.47
Temporal forecasts			65.65	37.00	50.39	8.33	1.15
Spatiotemporal forecasts			65.48	36.97	50.44	8.38	1.02
Provisional results	**10.01**	313	62.99	45.36	41.33	9.22	17.68
Spatial forecasts			63.63	38.65	47.94	8.90	4.68
Temporal forecasts			65.72	36.79	50.56	8.33	.77
Spatiotemporal forecasts			65.50	36.87	50.54	8.43	.77
Provisional results	**14.99**	473	63.50	44.60	42.02	9.22	16.23
Spatial forecasts			64.32	38.22	48.83	8.75	3.51
Temporal forecasts			65.80	37.03	50.13	8.49	1.28
Spatiotemporal forecasts			65.71	36.92	50.46	8.37	.96
Provisional results	**25.01**	793	63.84	44.07	42.53	9.27	15.14
Spatial forecasts			64.77	37.76	49.08	8.79	2.76
Temporal forecasts			65.84	36.93	50.34	8.42	1.04
Spatiotemporal forecasts			65.82	36.85	50.50	8.43	.79

NOTE: Calculated using data from ICM (2003a, 2003b).

[a] The result lines show the proportions of valid votes that each political party was receiving at the time point of interest. The forecast lines portray the final votes proportion forecasts obtained after applying the corresponding strategy.

[b] The percentage of census polled at each time point appears in the result rows of this column.

[c] The number of stations polled at each time point appears in the result rows of this column.

[d] The result lines show the voters' participation rate at that time point. The forecast lines portray the final participation rate forecasts obtained after applying the corresponding strategy.

[e] The error column displays the sum of the differences in absolute values between final outcomes and provisional data (either results or predictions), $\sum_j |z_j - \hat{z}_j|$.

government. For the first time in the Spanish democracy, a major election had to be repeated. The question for this new election was which party would be blamed by voters. (There were rumors that the accord had been blocked by people close to the PP.) The outcome went the same way as the previous election (e.g., with 50% of votes polled, PS + IU had 59 seats, compared with 52 for PP) and was uncertain throughout almost the entire elections night; it was not until 95% of the ballots had been counted that, for the first time, PP surpassed PS + IU.

Table 3 presents the OK, HD, and OCK forecasts for the October 2003 Asamblea de Madrid election in the city of Madrid (2,343,899 voters in 3,190 stations). The table shows results similar to those found previously: Spatial and spatiotemporal forecasts notably improve provisional outcomes, spatiotemporal are always better than spatial estimates, and temporal and spatiotemporal forecasts produce comparable error scores, although spatiotemporal figures dominate when the number of counted stations increases. Nevertheless, in contrast to preceding examples, the spatial strategy had difficulty in reducing error levels; one-quarter of the votes had to be polled before the error was <3%. This is likely a consequence of extremely biased provisional results (wrong by >15%).

5. CONCLUSIONS

Analyzing the geography of almost every election shows that outcomes display spatial dependence. Clear patterns can be identified in the spatial distribution of votes. Despite this fact, we know of no election forecasting method proposed in the literature that uses this dependence to improve predictions. To fill this gap, we have explored how introducing the geography of electoral data into this problem can affect the quality of the forecasts. In particular, two geostatistical techniques—ordinary kriging and ordinary cokriging—are used to estimate, in the context of election night forecasting and using polling stations as the spatial locations, the share of votes reached by the main parties in three elections for which conventional forecasting methods had great difficulty (because of slow convergence of the incoming votes to the final results and marked swings among parties). To gauge the value of spatial predictions, we also have developed a third group of forecasts based on linear forecasts with purely temporal data.

The results demonstrate that both spatial (kriging) and spatiotemporal (cokriging) forecasts strongly improve provisional outcomes. Nevertheless, spatial estimates seem to be more sensitive to provisional results and to require data from more locations to reduce their error levels. As a rule, spatiotemporal predictions are better than spatial estimates. Compared with the temporal forecasts, spatiotemporal estimates from early returns show quite similar errors. But spatiotemporal predictions systematically improve over temporal forecasts as soon as $>5\%$ of the votes are counted (for our examples). Indeed, even spatial forecasts obtain better results than temporal forecasts when the number of polled stations is large and their locations are dispersed.

We believe that spatial strategies represent a superior alternative for making election night predictions. Indeed, assisted by the increasing availability of geographical information systems, we hope that in the future other spatial methods, like spatiotemporal regression models (Kyriakidis and Journel 1999) or fragmentation statistics (Lucieer 2007) will be used to explore this problem. Furthermore, other statistical techniques, such as optimum paths or routing problems (Stevens and Olsen 2004), could be combined with spatial data mining to improve accuracy and to reduce costs in survey polls.

APPENDIX: AGGREGATION OF POLLING STATION PROPORTION OF VOTES

Let v_{kj} be the total number of votes assigned to party k in polling station j. It is easy to prove that

$$\begin{aligned} z_k &= \frac{\sum_{j=1}^{s} v_{kj}}{\sum_{j=1}^{s} n_j} = \frac{\sum_{j=1}^{s} n_j x_{kj}}{\sum_{j=1}^{s} n_j} \\ &= \frac{\sum_{j=1}^{s} \pi_j N_j x_{kj}}{\sum_{j=1}^{s} \pi_j N_j} = \frac{\sum_{j=1}^{s} \pi_j \omega_j x_{kj}}{\sum_{j=1}^{s} \pi_j \omega_j} \\ &= \frac{\sum_{j=1}^{s(t)} \pi_j \omega_j x_{kj} + \sum_{j=s(t)+1}^{s} \pi_j \omega_j x_{kj}}{\sum_{j=1}^{s(t)} \pi_j \omega_j + \sum_{j=s(t)+1}^{s} \pi_j \omega_j}. \end{aligned} \tag{A.1}$$

At a given moment t, we obtain an estimate for z_k by replacing the unknown proportion of votes (x_{kj}) in (A.1) with their approximations ($\hat{x}_{kj}$). At time t, however, the values n_j (the number of votes registered in station j), and thus the rates π_j, also are unknown for $j > s(t)$. Therefore, to compute this equation it is necessary to substitute estimates of π_j. As a result, eq. (1) is obtained.

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