Abstract
Some engineers are skeptical of statistical, let alone geostatistical, methods. Geostatistical analysis in reservoir characterization necessitates an understanding of a new and often unintuitive vocabulary. Statistical approaches for measuring uncertainty in reservoirs is indeed a rapidly growing part of the best-practice set of methodologies for many companies. For those already familiar with the basic concepts of geostatistics, it is hoped that this overview will be a useful refresher and perhaps clarify some concepts. For others, this overview is intended to provide a basic understanding and a new level of comfort with a technology that may be useful to them in the very near future.

Introduction
Geoscientists and geological engineers have been making maps of the subsurface since the late 18th century. The evolution of our ability to predict structure beneath the surface of the Earth has been a complex interaction between quantitative analysis and qualitative judgment. Geostatistics combines the empirical conceptual ideas that are implicitly subject to degrees of uncertainty with the rigor of mathematics and formal statistical analysis. It has found its way into the field of reservoir characterization and dynamic flow simulation for a variety of reasons including its ability to successfully analyze and integrate different types of data, provide meaningful results for model building, and quantitatively assess uncertainty for risk management. Additionally, from a management point of view, its methodologies are applicable for both geoscientists and engineers, thereby lending itself to a shared Earth model and a multidisciplinary workforce.

Why Geostatistics?
Fig. 1 depicts two images of hypothetical 2D distribution patterns of porosity. Fig. 1a shows a random distribution of porosity values, while Fig. 1b is highly organized, showing a preferred northwest/southeast direction of continuity. While this difference is obvious to the eye, the classical descriptive-summary statistics suggest that the two images are the same. That is, the number of red, green, yellow, and blue pixels in each image is the same, as are the univariate statistical summaries such as the mean, median, mode, variance, and standard deviation (Fig. 1c). Intuitively, as scientists and engineers dealing with Earth properties, we know that the geological features of reservoirs are not randomly distributed in a spatial context. The reservoirs are heterogeneous and have directions of continuity in both 2D and 3D space and are products of specific depositional, structural, and diagenetic histories. Strangely, that these two images would appear identical in a classical statistical analysis is the basis of a fundamental problem inherent in all sciences dealing with spatially organized data. Classical statistical analysis inadequately describes phenomena that are both spatially continuous and heterogeneous. Thus, use of classical statistical descriptors alone to help characterize petroleum reservoirs often will result in an unsatisfactory model.

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Key Benefits to Petroleum Reservoir Engineering

It is important to state that a geostatistical approach to reservoir characterization and flow modeling is not always required and does not universally improve flow-modeling results. However, there are several benefits for reservoir engineers when it is deemed appropriate.

- The first and most obvious benefit is that the technology is numerically based. The products are numerical, thus addressing a traditional barrier faced when introducing qualitative geologic data (e.g., depositional facies) into a flow-simulation model. Therefore, the final product is a volume of key petrophysical properties constrained to a set of facies that realistically depict the geological conceptual model and honor the well and seismic data. This result by itself can reduce the time required for effective history matching during flow simulation. Further, it is possible for the geostatistical static model to honor well tests and production data, which also reduces the time for history matching.

- Another benefit is in the shared responsibility for the construction of the static model, the input to the flow simulator. Not too many years ago, the static model was constructed by the engineer. That is, the integration of the geological, petrophysical, and geophysical data was performed by the engineer in preparation for flow simulation. This integration required a relentless and often unsuccessful effort to understand both conceptually and numerically the input data from each discipline. A particularly troublesome area was to understand the importance of the conceptual geological model because it required knowledge of the distribution of petrophysical properties and of depositional systems. Today, the responsibility of delivering a useful numerical model incorporating descriptive geological information rests on a group of domain experts constituting a team, rather than one individual.

- Geostatistically based static models that characterize reservoir heterogeneities are capable of improving the mechanistic understanding of fluid flow. This understanding is particularly true in complex heterogeneous situations, such as the presence of strong contrasts in permeability at multiple scales. Further, geostatistics has improved flow-simulation technology by challenging some of the theoretical foundations behind flow simulators (King and Mansfield 1999).

- Generally, petroleum engineers find it easier to quantify the variation in production forecasts by use of a geostatistical-simulation model to describe heterogeneities. Selecting extreme images that are based on simplified fluid-flow simulations (like streamlines) is acceptable for water cuts (mobility ratios up to 100), hydrocarbon production before breakthrough, identifying poorly swept regions (qualitative analysis of flow behavior), and studying the production forecasts from each image (realization) to quantify the range of possible hydrocarbon production (Guerillot and Morelon 1992).

This paper attempts to articulate both the practical use and common misconceptions of geostatistics applied to petroleum-reservoir characterization. Given the mandate for brevity, the material discussed here is presented more as an “armchair” discussion than as a formal synthesis of geostatistical principles. Thus, the detailed mathematics and formal statistical underpinnings are not presented. This material is presented in the context of a typical reservoir-characterization workflow broadly covering five basic steps shown in Fig. 2: (1) exploratory-data analysis (EDA), (2) spatial modeling, (3) kriging, (4) conditional simulation, and (5) uncertainty analysis. Scaling up, the process of coarsening the final high-resolution geological model in preparation for flow simulation, is not discussed.

Data Analysis

While EDA is not specifically geostatistical, it is a prerequisite for ensuring data integrity and is the first critical step in reservoir modeling. EDA consists of scrutinizing the data for errors, calculating the descriptive statistics (univariate statistics) for each variable, and identifying how the variables relate to one another (multivariate statistics). Performing EDA requires that the data be digital and placed in the context of a conceptual geological model.

These tasks are not trivial, and experience has shown that between 50 and 75% of the total time allocated to a reservoir-characterization project is consumed by preparing the data. Further, in polling hundreds of participants from reservoir-modeling classes and reviewing dozens of projects worldwide, it can be stated confidently that the bulk of this time is spent finding and cleaning the data before analysis. Why so much time is spent in this task and how it can be streamlined is beyond the scope of this paper. However, the effort involved is crucial if a reliable reservoir model is to be constructed. Managers and project leaders would do well to assess the allocated time for EDA accurately in their reservoir-modeling efforts, because it is often given too little time.
To construct a 3D reservoir model, all the data from the contributing disciplines are brought together and subjected to various mathematical formulas that use a variety of statistical metrics. EDA can be categorized into four basic steps: univariate analysis, multivariate analysis, data transformation, and discretization. Univariate analysis consists of profiling the data by calculating such traditional descriptors as the mean, mode, median, and standard deviation, to name a few. Multivariate analysis consists of examining the relationship between two or more variables with methods such as linear or multiple regression, the correlation coefficient, cluster analysis, discriminant analysis, or principle-component analysis. Data transformations are used for placing data temporarily into a convenient form for certain types of analyses. For example, permeability often is transformed into logarithmic space as a convenient way of relating it to porosity. Geostatistical analyses, such as conditional simulation, require data to be transformed temporarily into standard normal space to honor the assumption of normality implicit in the algorithms. All transformations require a precise back-transform that returns the data to their initial state. Typically, the standard normal transformation is used in geostatistical analyses (Deutsch and Journel 1997). Discretization is the process of coarsening or blocking data into layers consistent within a sequence-stratigraphic framework. Original data, such as well-log or core properties, are resampled into this space. The process is not trivial and must be checked against the raw-data statistics to ensure preservation of key heterogeneities (Fig. 3).

**Spatial Modeling**

Geological features and associated petrophysical properties generally are not distributed isotropically within a depositional environment. This fundamental principle generally is not addressed well in most computer-based interpolation algorithms, and it is the basis for early complaints about computer mapping. Geostatistics provides a method for identifying and quantifying anisotropic behavior in data with metrics that are used during interpolation or simulation to preserve directions and scales of continuity. The method is called variography, and the set of metrics it produces is identified from a graph called the semivariogram (hereinafter referred to simply as the variogram).

To understand variography, it is important to describe how interpolation algorithms use control points to estimate a value at a grid node or, more generally, an unsampled location. Most interpolation algorithms require two basic inputs: a grid and a set of control points (wells). Estimates can be provided at grid-cell centers or at the grid nodes, commonly referred to as “cell centered” and “corner point” estimation, respectively. If the grid cell is in 3D, then the cells are referred to as “voxels.” The advantages and disadvantages of each method are beyond the scope of this paper. For more information on gridding methods, see Lyche and Schumaker (1989). To compute an estimate at a specific unsampled location, the algorithm searches for nearby control points within a “neighborhood.” There are several parameters that can be set to control the search neighborhood, but, ultimately, a set of neighbors is selected. While an estimated value could be calculated easily by a simple arithmetic average of the neighboring values, most algorithms will weight the neighboring control points inversely by distance. That is, neighbors that are far from the unsampled locations to be estimated are given lower weights than those that are close by. The technique is known as inverse-distance weighting (Davis 1986). While the concept seems to be reasonable, it is flawed in...
that it assumes that the weight applied to a given neighbor at a specified distance is the same regardless of its azimuthal position from the unsampled location. As shown in Fig. 4, the artifact resulting from isotropic weighting is commonly expressed as contours that bend toward one another, forming a “figure 8” shape with the narrow waist perpendicular to the major axis of continuity. A weighting scheme is needed that embraces the concept of distance weighting while simultaneously considering anisotropy. Variography offers such a solution to this distance- and directional-weighting problem.

The variogram is computed by analyzing sample pairs. Sample pairs can be well-log data, cores, or seismic common-depth-point data. They can be from continuous variables like porosity or density, or discrete variables like geologic facies. The variogram can be computed in any direction: horizontally and vertically. The concept is to compare pairs of data values at a variety of regular separation distances, known as “lags.” The measured values from each sample (e.g., structural elevation, porosity, and permeability) in a given pair are subtracted from one another, and the result is squared to ensure that the number is positive. One could surmise that a pair of measurements very close together would have a squared difference close to zero and that measurements at increasingly larger separation distances would have increasingly larger squared differences. However, when separation intervals reach a certain distance, there is no longer any expectation that the values will follow any regular behavior. A single pair of points at a large lag can have one very low and one very high value or two values close in magnitude. Any similarity or dissimilarity would be completely random. If the results for each pair in a given lag are summed and averaged, then plotted as the mean squared difference (variance) against the mean lag distance, the experimental (semi)variogram is produced. If all possible pairs are used irrespective of azimuth, the result is known as an “omnidirectional” variogram. If pairs are selected such that they have a particular orientation, the results are known as “directional” variograms. A typical variogram shape is shown in Fig. 5.

The variogram shape is predictable, and its attributes must be recorded and used later in modeling. The required modeling components of the variogram are shown in Fig. 5 and consist of the “sill,” “range,” and “nugget.” The inflection point at which the variogram flattens is called the “sill” and is theoretically equal to the true variance of the data. The distance at which the sill is reached is called the “correlation range or scale” and defines the distances over which there is a predictable relationship with variance. Beyond the inflection point, the data are not correlated, and no predictable relationship can be defined. The “nugget effect” occurs when the slope of the variogram intersects the y-axis above the origin, suggesting the presence of random or uncorrelated “noise” at all distances. Often, the nugget is the result of sample aliasing where the geological feature of interest occurs at a scale smaller than the sampling interval (i.e., well spacing). As discussed later, modeling with or without a nugget can have a significant effect on mapping.

As mentioned earlier, one goal is a spatial weighting scheme. By constructing directional variograms, the changes in correlation range with azimuth can be observed. For example, imagine an anticline oriented northwest/southeast. A variogram constructed of pairs at each lag oriented in the northwest/southeast direction should show a correlation range that is considerably longer than that in the northeast/southwest direction, perpendicular to the strike of the anticline. Fig. 6a shows such a variogram. In practice, determining the minimum and maximum directions of continuity is tedious, requiring construction of many variograms with a variety of azimuths. Fortunately, most geostatistical packages now offer a simpler solution by computing the variogram “map,” which depicts variance for all azimuths (Fig. 6b) (Davis 1986). The reader is cautioned in interpreting the word “map.” Here, it is not a geographic map, but rather a polar graph of variance and azimuth along lag increments.

The final step in variography is to model the experimental variogram. Modeling is necessary because the experimental
variogram reports the variance only at the centroid of each lag interval. The kriging and simulation algorithms require knowledge of the variance at all possible distances and azimuths. Modeling the variogram is not a curve-fitting exercise in the least-squares sense (Gringarten and Deutsch 1999). The goal of the modeling exercise is to capture the sill, slope, range, and nugget (if present) by use of a specific set of functions, usually the spherical, exponential, Gaussian, linear, or power function. Only these functions, along with a finite set of others, are authorized models that ensure stability in the mathematics (Isaaks and Srivastava 1989).

The variogram contains much information that is beyond the scope of this paper. The value of the sill and range, the presence or absence of a nugget, the steepness of the slope, the nuances of its shape at different lags, and the various patterns seen on variogram maps all contribute to a deeper understanding of the data.

**Kriging**

With the data subjected to EDA, and the spatial model (variogram) constructed, the next step is to interpolate the key variables for the reservoir characterization onto a grid. The interpolation method used in geostatistics is kriging. The reader is referred to Isaaks and Srivastava (1989) for a discussion of the kriging matrix. **Fig. 7** compares three maps of porosity from a west Texas oil field that has a known location.

**Fig. 6**—Bidirectional variogram (a) shows the minimum and maximum directions of continuity. Variogram “map” (b) shows all directions of continuity simultaneously—variance increases from light blue (low) to brown (high). The orientation of the directional ellipse (N10°W), representing the anticline, is well defined, making selection of the maximum and minimum direction of continuity simple.

**Fig. 7**—Interpolation comparison of a porosity surface for a west Texas oil field containing 55 wells. Shown are (a) a typical isotropic inverse-distance-based method, (b) kriging with an omnidirectional variogram, and (c) kriging with a directional variogram. Note the increased degree of continuity in the north-northeast/south-southwest direction in (c).
direction of continuity roughly north/south. Fig. 7a uses a standard inverse-distance interpolation algorithm, Fig. 7b uses kriging with an omnidirectional variogram, and Fig. 7c uses kriging with a north/south directional variogram. Note the difference in general continuity, particularly when using the directional variogram.

A major advantage of kriging over other interpolation algorithms is the ability to use more than one variable simultaneously to predict the value at an unsampled location. The procedure is the multivariate case of kriging called “collocated cokriging.” The most common use of collocated cokriging is combining well data with seismic data to interpolate a structural surface, or combining seismic acoustic impedance and porosity measurements from wells to predict porosity. Fig. 8 depicts a series of collocated cokriged results from different degrees of correlation between acoustic impedance and porosity. The kriged porosity map takes on more of the seismic character as the correlation increases. When the correlation between acoustic impedance and porosity is low, the results resemble the kriging solution without using the seismic covariable. Note, however, that the method is not equivalent to a simple linear-regression rescaling. A full discussion of collocated cokriging can be found in Goovearts (1997).

**Conditional Simulation: Capturing Heterogeneity**

Conditional simulation provides engineers and geoscientists with the ability to produce practical reservoir models that reflect the proper spatial relationships among the various geological elements and their petrophysical properties as well as the heterogeneous nature of those properties. Further, the results can be expressed in probabilistic terms, allowing quantification of uncertainty and providing valuable input to flow simulation and risk analysis.

Fig. 9 compares two interpolation methods with conditional simulation. Fig. 9a is a cross section through a hypothetical sand/shale sequence and serves as the reference image. This cross section was sampled at three locations, labeled 1, 2, and 3, and the sample data were used to construct the remaining images. Figs. 9b and 9c are the result of interpolation: inverse distance and kriging, respectively. Kriging and inverse distance are similar with respect to preserving only the low-
Because all interpolation algorithms are fundamentally averaging techniques, they produce results that are inherently smoother than reality. The only practical difference between Fig. 9b and 9c is that kriging tends to honor the input data better. Figs. 9d and 9e resemble the reference image more closely by capturing the high-frequency information while honoring the input data. These more-heterogeneous-looking images represent two of many possible solutions (realizations) produced by use of a conditional-simulation algorithm. Like kriging, conditional simulation honors the well data but produces results that differ in the interwell space. The degree of difference is a function of the amount and spacing of data as well as their inherent variability. Each realization is said to be equally probable.

The preservation of the high-frequency component is a proxy for heterogeneity and is what makes conditional-simulation methods appealing to many reservoir modelers. Conditional simulation can be used in various stages of the modeling effort. Pixel-based methods operate on one pixel at a time to simulate structural surfaces, isopachs, and petrophysical properties, for example. They are distinguished from object-based methods (also referred to as Boolean) that operate simultaneously on groups of connected pixels to create geological objects, and they are used exclusively for simulating facies. Typical pixel-based methods include turning-bands simulation (Mantoglou and Wilson 1982), sequential Gaussian simulation (Deutsch and Journel 1997; Goovaerts 1997; Lantuejoul 1993; Lantuejoul 1997; Matheron 1975).

There are several conditional-simulation algorithms, and vendors have incorporated some of them into commonly used modeling packages. In the space allotted for this paper, it is impossible to discuss the details of each simulation algorithm. Instead, a more general discussion is offered that attempts to compare some of the basic differences among the algorithms and offer some guidelines for selecting a specific type of algorithm (Yarus et al. 2002).

Pixel-Based and Object-Based Simulation. Conditional simulation can be used in various stages of the modeling effort. Pixel-based methods operate on one pixel at a time to simulate structural surfaces, isopachs, and petrophysical properties, for example. They are distinguished from object-based methods (also referred to as Boolean) that operate simultaneously on groups of connected pixels to create geological objects, and they are used exclusively for simulating facies. Typical pixel-based methods include turning-bands simulation (Mantoglou and Wilson 1982), sequential Gaussian simulation (Deutsch and Journel 1997; Goovaerts
1997; Deutsch 2002), probability-field simulation (Deutsch and Journel 1997; Goovearts 1997; Deutsch 2002), and truncated Gaussian simulation (Galli et al. 1994). Object-based methods consist of variations of the general marked-point process (Lia et al. 1997; Hastings 1970) (Fig. 10). There are various implementations of all these algorithms, including multivariate forms that allow for the integration of secondary data such as seismic or trend maps. Particular attention should be paid to the parameterization of these algorithms to ensure their proper use.

Fig. 11 shows the results of facies simulation and subsequent petrophysical simulation of a west Texas oil field. Facies simulations for two different layers in the reservoir were constructed with the two genres of simulation algorithms: object-based (Fig. 11a) and pixel-based (Fig. 11b). Figs. 11c and 11d demonstrate the respective petrophysical simulations.

**Uncertainty Analysis**

Conditional simulation results in numerous realizations, each somewhat different from the next, depending on the amount of data available and the degree to which the reservoir is truly understood. The reason for generating multiple realizations is to enable quantitative assessment of the degree of uncertainty in the model being built. It is through these realizations that the “stochastic” characteristics become known. The degree of difference from one realization to the next is a measure of the uncertainty. Summarizing the realizations into a set of statistical metrics and displays uses the full potential of this method.

There are a few important statistical summaries and summary maps to consider when evaluating a set of realiza-
tions—in particular, the mean and standard deviation of the realizations and the integrated probability-distribution curve. The mean and standard deviation of the realizations can be derived from post-processing and provide the ability to make an estimate of a value at an unsampled location as well as the tolerance around it. Integrated probability curves can be generated for economic variables, such as stocktank oil originally in place or pore volume, then specific realizations that match key economic quantiles (e.g., P10, P50, and P90) can be used in flow simulations to estimate the possible, probable, and proven reserves (Fig. 12).

Post-processing realizations are key to understanding the bounds of uncertainty in developing a geostatistical reservoir-characterization model. The results have a wide range of applicability including development drilling, secondary and tertiary recovery, and, more broadly, portfolio analysis. There is a great deal more to uncertainty analysis, not the least of which includes various ranking strategies and the pooling of scenarios and realizations. These, however, will be left for future discussion. For now, this overview should leave the reader with the feeling that the process of applying geostatistical methodologies to reservoir modeling is a key component in defining the spatial continuity and capturing the space of uncertainty.

Concluding Thoughts and Future Directions

There have been several significant accomplishments, which are directly attributed to geostatistics, for describing reservoirs. In addition to providing a common technological framework accepted by most of the geoscience and engineering community, the most common achievements agreed upon by leading practitioners and researchers surveyed for this publication are the ability to integrate scaleable data, quantitatively describe heterogeneity, and capture the key range of uncertainty. Although some experts believe that the geostatistical applications in commonly used software packages have stagnated, some new methodologies are emerging. Of particular note are plurigaussian simulation (Armstrong et al. 2003) (currently available in some products) and multiple-point statistics (Caers and Zhang 2002; Strebelle 2005) (prototypes are available). Both of these powerful algorithms have the look and feel of Boolean simulation but overcome issues of high well density and traditional variography (Fig. 13).

Although there have been major advances over the past 2 decades, there remain definite areas for improvement. Perhaps the biggest nemesis plaguing geostatistics is its reputation of being a trade craft, and that it requires skilled experts. There is no denying that geostatistics cannot be applied haphazardly. It may be highly automated in some commercial software products, but the user is not excused from taking the time to understand what lies beneath the push of a button. There are very few aspects of this technology that cannot be mastered sufficiently through a series of professional seminars and through practice. The barrier we observe is not the inability of professionals to understand the mathematics, but the limited time given them to enhance their skill sets.

The next-most-common problems identified in discussions with practitioners deal with misunderstandings and misconceptions believed prevalent within the general-user community.

- Geostatistics will never account for the physical processes that created the underlying spatial distribution, which will always remain an interpretation made by a responsible professional. The best we can hope for is the ability to peer through the mass of erroneous and contradictory data and see some semblance of order.
- There is not a single geostatistical method or workflow that can be applied to every problem. Every problem is dif-
different. A unique combination of methods and an individualized workflow should be selected and designed to solve each particular problem.

- Geostatistical results will never reproduce nature exactly. It will provide us with “polite fictions” that appear to behave realistically enough to improve our decision making for a particular period of time.
- Geostatistics is a methodology and does not compensate for the use of inappropriate variables. Good variables must be selected to provide meaningful results to the reservoir simulator.
- Geostatistics will never be automatic, and it will never compensate for poor implementation. It solves a certain class of problems very well when applied properly, but it does not solve every problem.

Although geostatistics requires the application of statistics and mathematics, most importantly it necessitates good domain expertise from the team of geologists, geophysicists, petrophysicists, and engineers. The statistics and mathematicians, while not to be taken lightly, remain tools of the practitioners, who are obligated to be diligent in scrutinizing not only the modeling steps but also the results. It is all too simple to become sophisticated “button pushers” and lose sight of common sense.

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