Introduction

The principle of geostatistical filtering techniques is based on the decomposition of the variable of interest into independent factors. These factors can then be assigned to the real underlying phenomenon, i.e. geologically meaningful, other to artifacts directly linked to the acquisition procedure (Dubrule, 2003). The interpolation of the factors using kriging type methods is then immediate. When two data sets are available, the idea is that the "geological" component is the same, while the artifacts are different but not correlated. By interpolating the geological component, we take most of the combination of the data sets. The paper illustrates the benefit of using the proposed methodology for mapping the top of a near surface karstic reservoir by refraction surveying.

Geostatistical filtering techniques

A two-step procedure is carried out. Firstly, variogram modeling consists in fitting analytical models defined by a few parameters (range, sill, nugget effect) to the experimental variograms computed from the input data. Then, interpolation techniques can be applied, relying on the previous variogram model and geostatistical filtering (Chilès and Delfiner, 1999).
The geostatistical framework is appropriate to reveal different features of a spatial component by considering a model of random function combining different scales of variability. Each component or factor can then be estimated separately by a particular kriging. Factorial kriging builds spatial filters based on the decomposition of the variogram function. It is equivalent to work in the frequency domain with a spectral representation.

$$Z(x) = Y_0(x) + Y_1(x) + \ldots + Y_u(x) + \ldots + m$$

Let us consider the following decomposition of the stationary random function Z(x) with a variogram $\gamma(h)$:

$$\gamma(h) = \gamma_0(h) + \gamma_1(h) + \ldots + \gamma_u(h) + \ldots$$

Each factor $Y_u(x)$ is a zero-mean random function not correlated to the other factors with a variogram $\gamma_u(h)$:

$$Y_{u_0}^*(x_0) = \sum_{\alpha=1}^{n} \lambda_\alpha^{u_0} Z(x_\alpha)$$

Each factor can then be kriged from the data $Z(x_?)$ according to the following system:

$$\sum_{\beta=1}^{n} \lambda_\beta^{u_0} \gamma(x_\alpha - x_\beta) + \mu_{u_0} = \gamma_{u0}(x_\alpha - x_0) \quad for \quad \alpha = 1,\ldots,n$$

$$\sum_{\beta=1}^{n} \lambda_\beta^{u_0} = 0$$

The universality condition for the kriging weights to sum up to 0 guarantees the estimate has a zero-mean.
Subtracting the kriging of the factor Y_{u0} from Z(x) is the same as kriging Z(x) by filtering the component of variogram γ_{u0}. In that case of kriging with filtering the mean is incorporated to the variable and the sum of the weights is 1.
Applying the same idea to multivariate data sets leads to similar properties, instead of kriging, we have to use cokriging. The efficiency of the filtering technique depends on the interpretation of the decomposition into factors in relation with each variogram structure. A possible model considers that each variable is made of a common component (geologically meaningful) and not correlated residuals (artifacts).
Each variable is decomposed as follows:

$$Z_1(x) = G(x) + R_1(x)$$

$$Z_2(x) = G(x) + R_2(x)$$

where G(x) represents the repeatable part of the variables Z_i (the geological component) and $R_i(x)$ the residuals containing any remaining non repeatable effects such as acquisition artefacts and random noise.

That model has an important consequence on the bivariate variogram model (Coléou, 2002):

$$\gamma_{11}(h) = \gamma_G(h) + \gamma_{R_1}(h)$$

$$\gamma_{22}(h) = \gamma_G(h) + \gamma_{R_2}(h)$$

$$\gamma_{12}(h) = \gamma_G(h)$$

By considering that the residuals are independent, it results that the cross-variogram is the variogram of the geological component we want to interpolate. For each variable its simple variogram is obtained by adding to the cross-variogram the variogram component that will be filtered out.

Example of Refraction surveying

The dataset comes from the Experimental Hydrogeological Site (EHS), located near Poitiers city (France). The concerned aquifer consists of tight karstic Jurassic carbonates of about 100 m in thickness beneath a weathering zone of Tertiary clays.

Refraction seismic surveying, described in detail by Mari and Porel (2007), has been used to map the irregular shape of the top of karstic reservoir. The complete survey is composed of 20 receiver lines (in line direction) with a 15 m distance between adjacent lines. Each line is composed of 48 single geophones with 5 m between adjacent geophones. In the in line direction, a direct shot and a reverse shot have been recorded per receiver line. An additional shot point in the cross line direction has been fired at a distance of 60 m from the receiver line under consideration. The seismic implementation allows the acquisition of two sets of delay time curves from two different acquisition geometries (in-line shots and cross-line shots).

The following approach has been carried out on each data set independently (Jeannée, 2008):

- The experimental variograms computed in the main directions of continuity (in line and cross line directions). A map obtained by a kriging filtering the nugget effect and the small range structure put into evidence erroneous lines, one line on each data set. Removing these erroneous lines improves the correlation of variables (Figure 1).

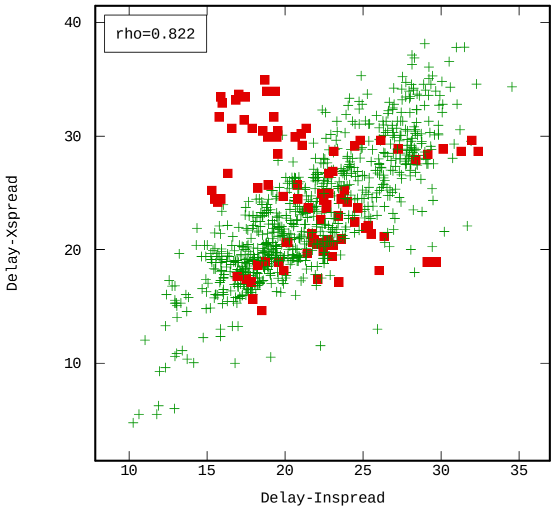

Figure 1: *Scatter Diagram of the two surveys (the red squares correspond to the erroneous lines).*

- After removal of these anomalous data a new variogram is calculated. The two surveys are interpolated independently on a grid 2.5mx5m by means of kriging with filtering of the small range components.We observe that the global correlation is unchanged (Figure 2) while we would expect to have improved it because of the removal of the artifacts.

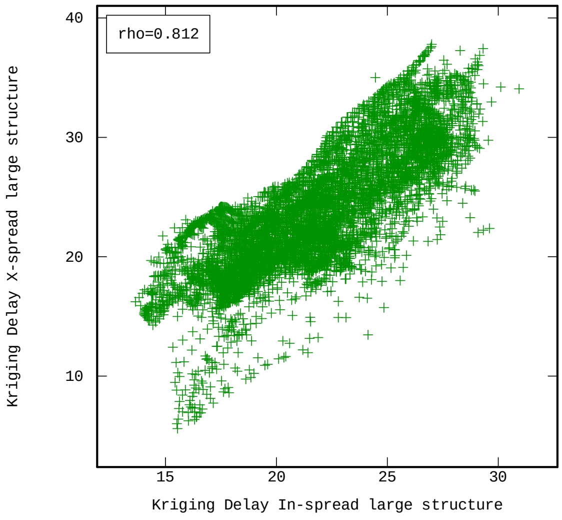

Figure 2: *Scatter Diagram of the two surveys after interpolation filtering the small range components.*

- The two data sets are now considered together. A bivariate variogram model is calculated based on the geological component obtained from the cross-variogram as explained above (Figure 3). The model is satisfactory at least up to a distance of 50m, which is enough because of the data density.

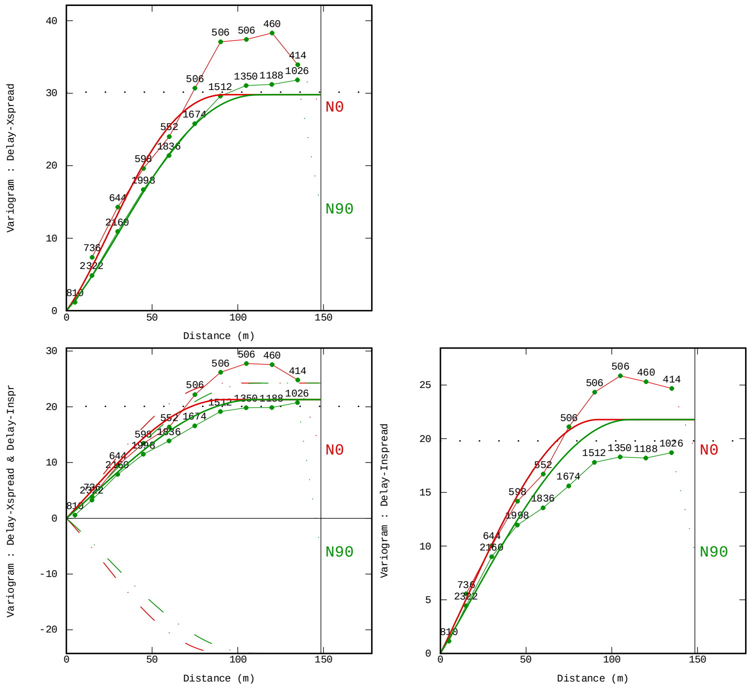

Figure 3: *Bivariate variograms of the two datasets after removal of the erroneous lines*

- The geological component is then interpolated by factorial cokriging. The 2 interpolated variables have now a coefficient of correlation of 1, which means that both variables are the same up to linear transform. Besides we can see (Figure 4) that cokriging method has preserved more of the initial variability than kriging.

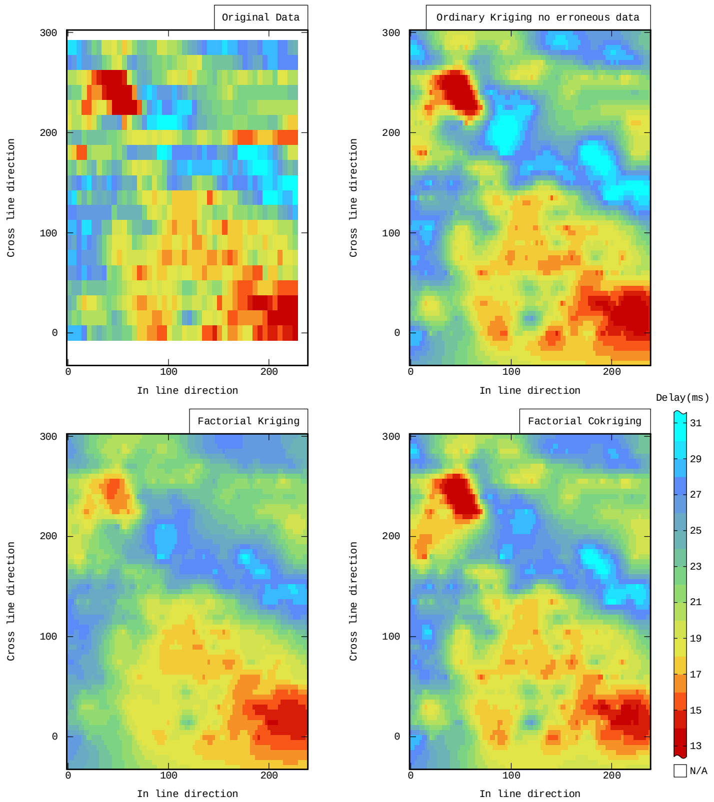

Figure 4: *detection of long wavelength anomaly from refraction survey. Example of the processing of first acquisition using factorial kriging and cokriging techniques.*

Conclusion

This work demonstrates how variogram analysis and geostatistical filtering can be used to detect and filter both random acquisition noise and foot prints in near-surface geophysics. It also illustrates how geostatistical filtering techniques can be used to merge two different surveys of the same area and optimize the use of the information for mapping the component making the common part of both data sets. The field example shows that an accurate delay map can be obtained with two sets of delay curves to map the top of a karstic reservoir and to point out corridor of fractures.

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References

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