MULTIVARIATE UNIFORM CONDITIONING AND BLOCK SIMULATIONS WITH DISCRETE GAUSSIAN MODEL: APPLICATION TO CHUQUICAMATA DEPOSIT.

JACQUES DERAISME[1], JACQUES RIVOIRARD[2] and PEDRO CARRASCO CASTELLI[3]
[1]*Geovariances, France*
[2]*Centre de Géosciences, Mines-ParisTech, France*
[3]*Codelco ,Casa Matriz, Chile*

ABSTRACT

Recoverable resources estimation has become a standard geostatistical application in the mining industry with different techniques being available. Improvements in the practical implementation of Uniform Conditioning and Direct Block simulations for multi-element deposits are presented. An application on copper and arsenic data of the Chuquicamata deposit illustrates the theoretical issues.

INTRODUCTION

Uniform Conditioning by panel grade consists of estimating the grade distribution on selective mining unit (SMU) support within a panel, conditional to the unique panel grade, usually an Ordinary Kriging to face a possible lack of stationarity. The general framework which forms the basis of Uniform Conditioning is the Discrete Gaussian Model of change of support, based in particular on the correlation between Gaussian-transformed variables. Recently mining companies have shown a renewed interest for Uniform Conditioning, because of its simplicity and efficiency in accounting for support and information effects at different scales. It was nevertheless restricted to selection applied to a main grade and recovery of the same element. The model has been extended to the multivariate case, where the correlations between all variables on any support can be calculated after transformation into gaussian space. In addition a rigorous formulation of the information effect on panel grade has been developed, that allows taking into account the heterogeneity of the data configurations when estimating panel grades.

The uniform conditioning provides results assuming a free selection of SMUs within panels and does not give any indication on the confidence level of the estimates. The above limitations justify resorting to simulations that can be used as input for making statistics on expected fluctuations and for simulating a selection processes that is closer to real mining operations. An important drawback is the prohibitive time-consuming algorithm for block simulations on large deposits, when these were obtained by averaging simulated values at points that discretize the blocks/SMUs. An alternative method for calculating the change of support coefficient due to Emery and Ortiz (2005) authorizes to perform the multivariate variographic analysis on Gaussian data and the application of a change of support model to get the block variogram model to be used for directly simulating the blocks/SMUs.

MODELS FOR NON LINEAR GEOSTATISTICS

Basis of the Discrete Gaussian Model

Let v be the generic selection block (SMU) and Z(v) its grade, that will be used for the selection at the future time of exploitation (we assume that this grade will be then perfectly known, i.e. there is no information effect on this). The recoverable resources above cutoff grade z for such blocks are:

- the ore T(z) = $1_{Z(v)\geq z}$
- the metal Q(z) = $Z(v)1_{Z(v)\geq z}$

We use here the discrete Gaussian model for change of support (e.g. Rivoirard, 1994). A standard Gaussian variable Y is associated to each raw variable Z . Let $Z(x)=\Phi(Y(x))$ be the sample point anamorphosis. The block model is defined by its block anamorphosis $Z(v)=\Phi_r(Y_v)$, given by the integral relation :

$$\Phi_r(y)=\int \Phi\ (ry+\sqrt{1-r^2}u)\,g(u)\,du \tag{1}$$

where the change of support coefficient r is obtained from from the variance of blocks.

Then the global resources at cutoff z are:

- ore: $E\left[T(z)\right]=E\left[1_{Z(v)\geq z}\right]=E\left[1_{Y_v\geq y}\right]=1-G(y)$ (2)
- metal: $E\left[Q(z)\right]=E\left[Z(v)1_{Z(v)\geq z}\right]=E\left[1_{Y_v\geq y}\Phi_r(Y_v)\right]=\int_y \Phi_r(u)\,g(u)\,du$ (3)

where g and G are the standard Gaussian p.d.f. and c.d.f., and y is the gaussian cutoff related to z through $z=\Phi_r(y)$.

Uniform Conditioning (UC) in the Univariate Case

UC by panel grade (Rivoirard, 1994) aims at estimating the recoverable resources on a generic selection block v uniform within a large block or panel V, conditioned on the sole panel grade, or for more generality, the panel grade estimate, say, Z(V)*.

$$\left[T_V(z)\right]^*=E\left[1_{Z(v)\geq z}\middle|Z(V)^*\right] \tag{4}$$

$$\left[Q_V(z)\right]^*=E\left[Z(v)1_{Z(v)\geq z}\middle|Z(V)^*\right] \tag{5}$$

The idea is to impose the panel grade, estimated for instance by Ordinary Kriging, in order to avoid the attraction to the mean that may be caused by some techniques in case of deviation from stationarity. The estimation of the metal at 0 cutoff must then satisfy the relation: E[Z(v) | Z(V)*] = Z(V)*. As v is uniform within V, it follows first that the panel grade estimate Z(V)* is implicitly assumed to be conditionally unbiased i.e. E[Z(V) | Z(V)*] = Z(V)*. Secondly it is assumed that the anamorphosis of Z(V)* is obtained from that of Z(v):

$$Z(V)^* = E\left[\Phi_r(Y_v) \mid Y_{V^*}\right] = \Phi_{r\rho_{vV^*}}(Y_{V^*}) = \Phi_S(Y_{V^*})$$

assuming the standard Gaussian variables Y_v and Y_{V^*} being bigaussian, and denoting $S = r\rho_{vV^*} = r\, corl(Y_v, Y_{V^*})$.

In practice S is obtained from the variance of the panel estimate. The previous relationship is used to compute the correlation between the block and the panel estimate: $corl(Y_v, Y_{V^*}) = \rho_{vV^*} = S / r$

The ore tonnage and metal at cutoff $z = \Phi_r(y)$ are then

$$\left[T_V(z)\right]^* = E\left[1_{Z(v)\geq z} \middle| Z(V)^*\right] = E\left[1_{Y_v \geq y} \middle| Y_{V^*}\right] = 1 - G(a) \quad (6)$$

$$\left[Q_V(z)\right]^* = \int_a \Phi_r(\rho_{vV^*} Y_{V^*} + \sqrt{1-(\rho_{vV^*})^2}\, u)\, g(u)\, du \text{ with } a = \frac{y - \rho_{vV^*} Y_{V^*}}{\sqrt{1-(\rho_{vV^*})^2}} \quad (7)$$

Uniform Conditioning in the Multivariate Case

Indices are now added to distinguish the variables. Let Z_1 be the metal grade used for the selection, and let Z_2 be one of the secondary metal grades. In addition to the univariate case seen above, we now wish to estimate the other metals, for instance:

$$Q_2(z) = Z_2(v) 1_{Z_1(v)\geq z}$$

Its global estimation is given by:

$$\begin{aligned} E\left[Q_2(z)\right] &= E\left[Z_2(v) 1_{Z_1(v)\geq z}\right] = E\left[1_{Z_1(v)\geq z} E\left[Z_2(v) \mid Z_1(v)\right]\right] \\ &= E\left[1_{Y_{1v}\geq y} E(\Phi_{2,\, r_2}(Y_{2v} \mid Y_{1v}))\right] = E\left[1_{Y_{1v}\geq y} \Phi_{2,\, r_2\rho_{1v2v}}(Y_{1v}))\right] \\ &= \int_y \Phi_{2,\, r_2\rho_{1v2v}}(u)\, g(u)\, du \end{aligned} \quad (8)$$

where r_2 is the change of support coefficient for Z_2, and Y_{1v} and Y_{2v} are bigaussian, with a correlation $corl(Y_{1v}, Y_{2v}) = \rho_{1v,2v}$ that is the covariance between both random variables.

Multivariate UC (Rivoirard, 1984) consists in estimating the recoverable resources of blocks v in panel V from the sole vector of panel estimates ($Z_1(V)^*$, $Z_2(V)^*$, …). The problem is simplified by assuming that:

- $Z_1(v)$ is conditionally independent of the auxiliary metal panel grades given $Z_1(V)^*$, and so the UC estimates for the selection variable correspond to the univariate case.
- similarly, $Z_2(v)$ is conditionally independent of $Z_1(V)^*$ given $Z_2(V)^*$,.
- $Z_1(v)$ and $Z_2(v)$ are, conditional, independent of the other metal panel grades given $(Z_1(V)^*, Z_2(V)^*)$. It follows that the multivariate case reduces to a bivariate case. In particular we have:

$$\left[Q_{2V}(z)\right]^* = E\left[Z_2(v) 1_{Z_1(v)\geq z} \middle| Z_1(V)^*, Z_2(V)^*\right] \quad (9)$$

We further impose, for the metal at cutoff 0:

$$E\left[Z_2(v) \mid Z_1(V)^*, Z_2(V)^*\right] = Z_2(V)^* = E\left[Z_2(v) \mid Z_2(V)^*\right] \quad (10)$$

This is similar to the univariate case, so that, first $Z_2(V)*$ is implicitly assumed to be conditionally unbiased. Secondly, the Gaussian anamorphosis of $Z_2(V)*$ is obtained from that of $Z_2(v)$ and is given by

$$Z_2(V)* = E\left[\Phi_{2,r_2}(Y_{2v}) \mid Y_{2V*}\right] = \Phi_{2,r_2\rho_{2v2V*}}(Y_{2V*}) = \Phi_{2,S_2}(Y_{2V*}) \tag{11}$$

The model is entirely specified by the anamorphosis, the different change of support coefficients, and the correlations between the Gaussian variables (Y_{1v}, Y_{1V*}, Y_{2v}, Y_{2V*}). The correlation between Y_{1v} and Y_{2v}, and that between Y_{1V*} and Y_{2V*} allow completing the correlations by using the conditional independence relationships:

- As $Z_2(v)$ and $Z_1(V)*$ are considered independent, conditional on $Z_2(V)*$, we have:

$$corl(Y_{2v}, Y_{1V*}) = corl(Y_{2v}, Y_{2V*})\, corl(Y_{1V*}, Y_{2V*})$$

$$\text{i.e. } \rho_{2v1V*} = \rho_{2v2V*}\rho_{1V*2V*} = \frac{S_2}{r_2}\rho_{1V*2V*} \tag{12}$$

- As $Z_1(v)$ and $Z_2(V)*$ are considered independent, conditionally on $Z_1(V)*$, we have similarly:

$$\rho_{1v2V*} = \rho_{1v1V*}\rho_{1V*2V*} = \frac{S_1}{r_1}\rho_{1V*2V*} \tag{13}$$

The secondary metal

$$[Q_{2V}(z)]^* = E\left[Z_2(v)1_{Z_1(v)\geq z} \middle| Z_1(V)^*, Z_2(V)^*\right] \tag{14}$$

can then be computed. After developments as detailed in Rivoirard (1984), we finally get the remarkable expression:

$$\left[Q_{2V}(z)\right]* = \int_a \Phi_{2,r_2 s_{2vK}}\left(\rho_{2vK2V*}Y_{2V*} + \varepsilon\sqrt{1-\left(\rho_{2vK2V*}\right)^2}u\right) g(u)\,du$$

with $\varepsilon = sign(r_{12})$, $\rho_{2vK2V*} = \dfrac{\rho_{2v2V*}}{s_{2vK}}$, and $(s_{2vK})^2 = (\rho_{2v2V*})^2 + \dfrac{{r_{12}}^2}{1-(\rho_{1v1V*})^2}$ (15)

Direct Block Simulations

The discrete gaussian model can be seen basically as a block model, where the domain is partitioned into small blocks v. Then each sample point is considered as random within its block, and conditional on its block value (here the multivariate value of the different elements), the point (multivariate) value does not depend on any other variable, whether they are values of other blocks or other points, even in the same block. This will allow to deduce the point-point and point-block covariances from the block-block covariances. After anamorphosis, all Gaussian values are considered as multi-gaussian, allowing conditional simulation. The model is specified by the simple and cross-covariances of block Gaussian variables, from which the other covariances can be deduced, for instance between elements 1 and 2 and blocks i and j:

- block-block cross-covariance: $Cov[Y_1,vi , Y_2,vj]$
- point-block cross-covariance: $Cov[Y_1(x_i), Y_2,vj] = r_1\ Cov[Y_1,vi , Y_2,vj]$
- point-point cross-covariance: $Cov[Y_1(x_i), Y_2(x_j)] = r_1 r_2\ Cov[Y_1,vi , Y_2,vj]$

except between the point and itself where the covariance is derived from the statistics on the data Cov[$Y_1(x_i)$, $Y_2(x_i)$] (for a single point element, the variance is 1).

The difficulty is to get a consistent model on Gaussian transformed variables from the model of the raw variables. The link between covariances on raw and gaussian variables is expressed by means of the gaussian anamorphosis model. But the uniqueness of the solution on inverting these relationships is not guaranteed. Another approach (Emery and Ortiz 2005) can be followed considering that the Gaussian transform on block support is the regularized point Gaussian variable, normalized by its variance, this being precisely the square of the change of support coefficient. This leads to another way of determining the change of support coefficient, using the variogram and variance of the gaussian variable, instead of those of the raw variable as used in the “classical” method:

$$r^2 = \operatorname{var} Y(v) = \operatorname{var} Y(x) - \overline{\gamma_Y(v,v)} \simeq 1 - \overline{\gamma_Y(v,v)}$$

In the same manner one can calculate directly the block gaussian covariances and cross-covariances from the regularized simple and cross-covariances of the gaussian data:

$$\operatorname{cov}(Y_{1v}, Y_{1v_h}) = \frac{\operatorname{cov}(Y_1(v), Y_1(v_h))}{r_1^2} = \frac{\overline{\rho_{Y_1}(v,v_h)}}{r_1^2} = \frac{1-\overline{\gamma_{Y_1}(v,v_h)}}{r_1^2} \quad (16)$$

$$\operatorname{cov}(Y_{1v}, Y_{2v_h}) = \frac{\operatorname{cov}\left[Y_1(v), Y_2(v_h)\right]}{r_1 r_2} = \frac{\overline{\rho_{Y_1Y_2}(v,v_h)}}{r_1 r_2} = \frac{\operatorname{cov}\left[Y_1(x), Y_2(x)\right] - \overline{\gamma_{Y_1Y_2}(v,v_h)}}{r_1 r_2} \quad (17)$$

CASE STUDY

Description of the Deposit

Geology

There are at least 3 arsenic mineralization events in this copper deposit. A younger mineralization pulse, related to the potasic alteration and at least two different later events are recognized that belong to the phyllic late-alteration stage. While in one stage enargite is in veins associated with pyrite, covellite, digenite, in the other late and more intense event, enargite occurs in high-grade veins, breccias and stocwork related to pyrite and, to a lesser extent, to tennantite. The present study is related to this last stage (Fig 1).

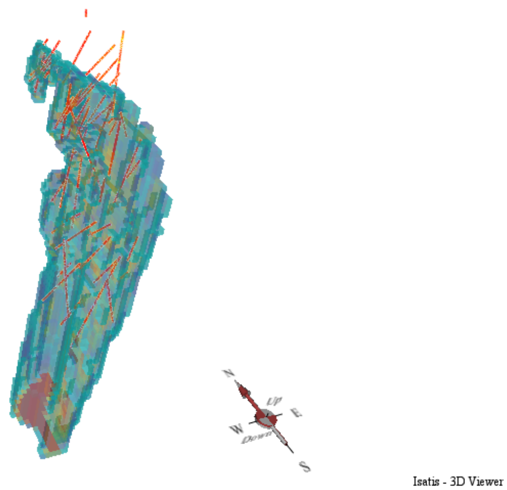

Figure 1: View of the high As area (color from pale blue to red depending on the ore proportion within 20x20x18m^3 blocks) and the layout of drillholes.

Data

We have considered the two variables Cu and As of the high As domain, composited on 3m. The data set, made of 68 drillholes, is almost perfectly isotopic (i.e. all variables sampled at all locations). The domain is defined on panels that have a square section of 20mx20m. The block model is made of regular panels of 18m high, which is the practical bench height. Each panel contains 6 smus of 20mx20mx3m. Uniform Conditioning requires estimating the panels. After cokriging, the statistics provide a more reliable estimate of the mean of the whole deposit (it is the consequence of the declustering property of kriging). The cokriging weights assigned to the Cut (total copper grade) composites when estimating Cut have been used for calculating the histograms as well as the corresponding anamorphosis and variograms. The table 1 shows that the declustering does not change the statistics for As.

Table 1: Statistics on the 3m composites before and after declustering (the difference in counts is due to exclusion of a few data with negative weights).

VARIABLE	Count	Minimum	Maximum	Mean	Std. Dev.	Variance
	2169	0.02	6.39	0.99	0.67	0.44
Cut	2127	0.02	6.39	0.82	0.47	0.22
	2165	0	2.11	0.22	0.16	0.02
As	2123	0	2.11	0.21	0.14	0.02

The linear coefficient of correlation of composites of As and Cut is 0.53, but it should be noted that the scatter plot are not elliptical and the regressions are not linear (Fig. 2). Things are not much different after the normal-score transform.

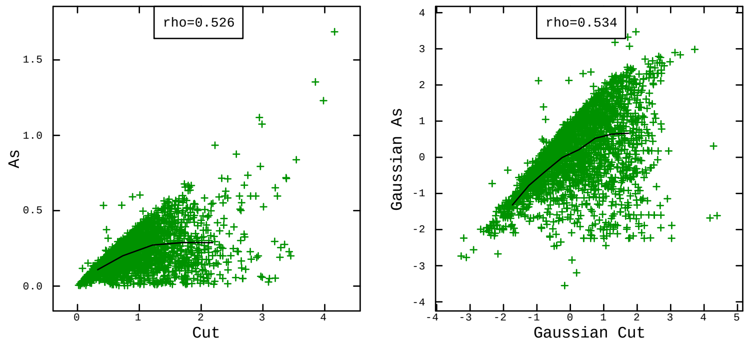

Figure 2: Scatter diagram of As and Cut on composites 3m with the regression curve (As|Cut) on raw data and after normal score transform.

Geostatistical Analysis and Modeling for Uniform Conditioning and Block Simulations

Variograms on composites

Uniform Conditioning requires the dispersion variances and covariance of the kriging of the secondary variable (As) and the main variable (Cut) on the panels. These statistics can be obtained from the multivariate variogram model, fitted on the weighted experimental variograms (Fig. 3). There is no evidence of anisotropy, and the variograms, computed up to 250 m, show a sill lower than the sample variance, due to a larger scale variability. We have considered the statistical variance as the most reliable estimate of the actual variance and have derived from it the dispersion variance of the SMUs.

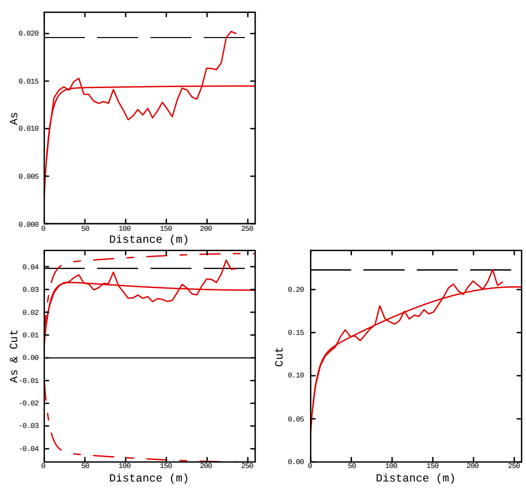

Figure 3: Bivariate experimental and modeled variograms on 3m composites

The weighted simple variograms on the normal score transforms have a sill closer to their variance of 1 (Fig. 4). The cross-variogram shows the existence of a structure at large distance (220m) with a negative correlation.

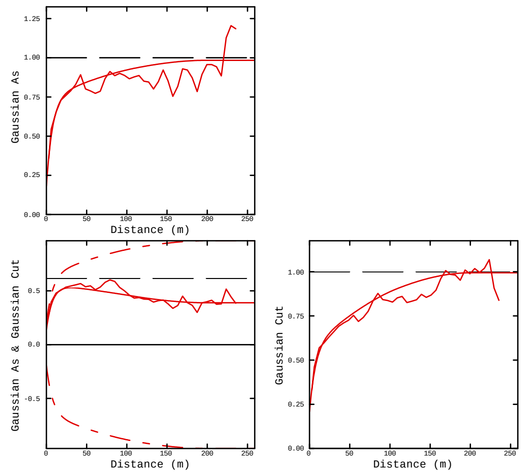

Figure 4: Bivariate experimental and modeled variograms on normal score transforms of 3m composites

Change of support

The change of support coefficients are calculated in two different ways for Uniform Conditioning and Direct block simulations.

The change of support used in Uniform Conditioning is calculated:

- for the SMU support, from the variance computed from each variogram model of raw data.
- for the kriged panels, from the theoretical dispersion variance of the cokriging of the panels. In order to account for the heterogeneity of the cokriging configurations, the panels can be classified according to the variance of the main variable, with a value of the change of support coefficient that depends on the class.

For the simulations the change of support coefficients result from the regularization of the Gaussian variogram model on the SMU support. Table 2 gives an idea of the variations of the coefficients according to the two approaches.

Table 2: Change of support coefficients for SMUs, calculated from the variogram of raw data (method 1) or from the variogram of Gaussian transforms. (method 2).

	Cut	As
Method 1	0.75	0.71
Method 2	0.71	0.64

Results

Comparison Between Uniform Conditioning and Direct Block Simulations

The grade tonnage curves when selecting on Cut (Fig. 5) show that the direct block simulation technique gives a slightly lower tonnage for cut-offs less than 0.8%. (and slightly higher values for cut-offs from 0.8 to 1.5 %) and slightly higher Cut grades than UC (the global grade-tonnage curves on SMU support is very close to the simulations).

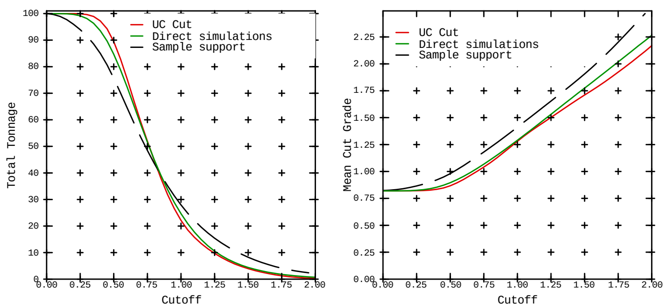

Figure 5: Grade-tonnage curves on smus Cut, calculated from UC or simulations

The average As grade when cutting on Cut is higher for UC, particularly at cut-off around 1.2 (figure 6).

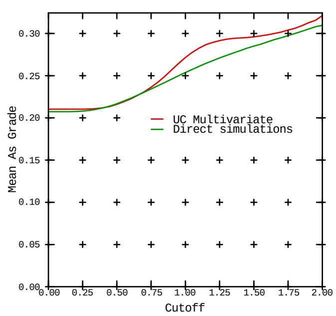

Figure 6: Mean As grade recovered after cutoff on SMUs Cut, from UC or simulations.

It should be noted that Uniform Conditioning and block simulations are very close for the range of practical cut-offs (between 0.25 and 0.5 % Cut). The difference in As mean grade appearing for higher cut-offs has applies to smaller tonnages and may come from a limited number of panels.

Comparison Between Simulations Obtained by the Direct and the Classical Methods

The block simulations have been averaged in order to calculate simulated panel grades. A further simulation post-processing has been carried out to average the 50 simulations (giving a approximation of the conditional expectation for each panel). The statistics on the 5308 panels entirely simulated (Table 3) show a good fit of the average grades obtained by cokriging and by simulations, and a higher standard deviation for cokriging.

Table 3: Statistics on the average of 50 simulated panel values obtained by direct block simulation compared with the panels cokriging values.

VARIABLE	Count	Minimum	Maximum	Mean	Std. Dev.
Cokriging Cut	5308	0.295	2.084	0.821	0.282
Mean of direct simulations of Cut 20x20x3	5308	0.259	1.99	0.82	0.23
Cokriging As	5308	0.043	0.78	0.21	0.07
Mean of direct simulations of As 20x20x3	5308	0.04	0.642	0.207	0.047

The classical simulation method consists in discretizing each block into n points and in averaging the simulated point values. The comparison of both methods has been made by averaging 50 simulations of panels, and shows behaviours that are largely similar, although there is a notable dispersion (Fig. 7).

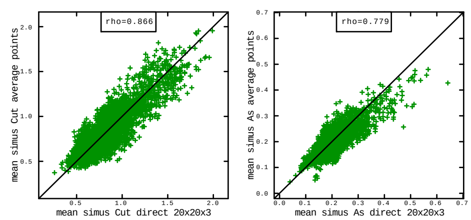

Figure 7: Scatter diagram of the average of 50 panels simulations obtained by averaging points or from the direct block simulations method**.**

CONCLUSIONS

The generalization of the discrete Gaussian model to the multivariate case has shown interesting applications for predicting recoverable resources that account for change of support. Regarding the sought objectives, these methods are remarkably simple, depending on a few key parameters, and can be run in a very short time.

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References

Emery, X. and Ortiz J. 2005, *Internal Consistency and Inference of Change-of-Support Isofactorial Models,* Geostatistics Banff 2004, Springer, 2005, p. 1057-1066.

Rivoirard, J. 1984, *Une méthode d'estimation du récupérable local multivariable,* Note 894, CGMM Fontainebleau, Mines Paris-Tech. 10 p. Edited under _________________________ .

Rivoirard, J. 1994, *Introduction to disjunctive kriging and non linear geostatistics*, Oxford, Clarendon Press. 181 p.