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Localised Uniform Conditioning in the Multivariate Case – An Application to a Porphyry Copper Gold Deposit

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ABSTRACT

The paper provides a brief review of the multivariate uniform conditioning and the localised multivariate uniform conditioning techniques and presents a case study based on a porphyry copper gold deposit in Peru. Two other important scenarios have been investigated and will be particularly emphasised:

1. The recovered selective mining unit (SMU)s depend on a combination of copper and gold content, which can be expressed by means of 'net smelter returns' (NSR). Two approaches are compared. Firstly, the application of a composite cut-off on NSR on the joint distribution of each grade and the application of uniform conditioning to three variables is investigated, ie for the first approach NSR is used as the main variable. The second approach considers copper as the main variable.
2. In the case of multiple domains the procedure used to apply the technique to process mixed panels is clarified and recommendations are made.

INTRODUCTION

For new mining projects or for medium and long-term areas of existing mines, drilling data are invariably on a relatively large grid. The general indirect estimation technique of recoverable resources during these planning phases derives the unknown SMU distribution estimates from the observed distribution of relative large kriged blocks (panels). The Gaussian model and the uniform conditioning (UC) technique provide an alternative consistent framework to achieve this task. This method may be extended to the multivariate case, ie for several multiple commodities related to the estimated recovered tonnage after cut-off are applied to the main economic element of the deposit.

The drawback of the indirect methods above is that only the probability distribution of the SMUs within local panels can be derived but not their individual spatial locations within the panel. Localised uniform conditioning (LUC) post-processing technique has been proposed to enhance the indirect uniform conditioning by localising the results at the SMUs scale.

The tonnages and metals represented by the grade tonnage curves estimated by indirect uniform conditioning are decomposed and distributed into the SMUs within respective panels according to a ranking of the main element grade estimate of the SMUs. As the correlations between the different grades and the main element are taken into account in the multivariate uniform conditioning (MUC), the local scale estimates for the other multiple commodity metals contained in the tonnage, which in this case is assumed to depend only on the main commodity grade, are immediately obtained from the metals attached to those tonnages (the resultant local SMU estimates are referred to as localised multivariate uniform condition (LMUC) estimates).

UC consists of estimating the grade distribution on SMU support within a panel, conditioned to the estimated panel grade, usually based on ordinary kriging (OK) or simple kriging (SK) with local mean to accommodate a possible lack of stationarity (ie when the average grade varies within the deposit). The general framework which forms the basis of uniform conditioning is the discrete Gaussian model of change of support, based in particular on the correlation between Gaussian-transformed variables. The mining industry's acceptance of the UC method has been observed for several years and a good

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reconciliation is generally found between UC medium- to long-term estimates and production data. The model has been extended to the multivariate case, where the correlations between main and secondary variables can be calculated on any support after transformation into Gaussian space. In addition, a rigorous formulation of the information effect on panel grades distribution has been developed, that allows taking into account the heterogeneity of the expected production data configurations when estimating future SMU recoverable grades.

The disadvantage of traditional indirect UC is that the outputs consist of panel local grade-tonnage curves representing a number of non-localised SMUs within these panels. Therefore, it is practically difficult to use these models for underground and open pit mine planning that requires a grade model on the SMU support. Abzalov (2006) proposed a solution using the grade tonnage functions from the indirect univariate UC and then decomposing the panel-specific grade-tonnage data into a suite of individual SMU sized units within the respective panels; according to a ranking of the main element grade estimate of the SMUs.

Assibey-Bonsu, Tolmay and Krige (2008) have extended Abzalov's proposal also in the univariate space particularly for underground mine planning. The direct approach referred to as localised direct conditioning (LDC) estimates by Assibey-Bonsu, Tolmay and Krige (2008) bases the grade tonnage recoverable functions on direct estimates of individual SMUs within the orebody (instead of indirect SMU distribution within panels).

This paper investigates and extends Abzolov's pr0posal to the multivariate UC case. As the correlations between the different grades and the main element are taken into account in the multivariate uniform conditioning, the local scale estimates for the other multiple commodity metals contained in the tonnage, which in this case is assumed to depend only on the main commodity grade, are immediately obtained from the metals attached to those tonnages. The correlations between the different elements on the SMU support are reproduced by means of the correlations introduced in the multivariate change of support model.

The authors present a brief review of the UC and LMUC techniques and a case study based on a porphyry copper gold deposit in Peru. The study also compares LMUC estimates for the SMUs when the cut-off grade is applied to total copper (CUTOT) as the main variable or on a combination of CUTOT and total gold (AUTOT), ie the latter is based on the block economic value, the net smelter returns (NSR). Thus, in the second scenario, NSR is used as the main variable instead of CUTOT. The limitation of the NSR option is that it is dependent on economic parameters that vary with time.

DISCRETE GAUSSIAN MODEL APPLIED TO RECOVERABLE RESOURCE ESTIMATION

Reminders on the discrete Gaussian model

Let 'v' be the generic SMU and Z (v) its grade, which will be used for the selection at the future production stage.

The recoverable resources above cut-off grade z for such blocks are:

$$\text{the ore } T(z) = 1_{Z(v) \geq z}$$

$$\text{the metal } Q(z) = Z(v) 1_{Z(v) \geq z}$$

where:

$1_{Z(v) \geq z}$ stands for the grade indicator at cut-off z, ie:

$1_{Z(v) \geq z} = 1 \; if \; Z(v) \geq z$; $1_{Z(v) \geq z} = o \; if \; Z(v) < z)$

We use here the discrete Gaussian model for change of support (Rivoirard, 1994). A standard Gaussian variable 'Y ' is associated with each raw variable 'Z'. Let $Z(x) = \phi(Y(x))$ be the sample point anamorphosis. The block model is defined by its block anamorphosis $Z(v) = \phi_r(Y_v)$, given by the integral relation:

$$\phi_r(y) = \int \phi\left(ry + \sqrt{1 - r^2 u}\right) g(u) du \qquad (1)$$

where the change of support coefficient r is obtained from the variance of blocks

Then, the global resources at cut-off z are:

Ore:

$$E[T(z)] = E[1_{Z(v) \geq z}] = E\left[1_{y_v \geq y}\right] = 1 - G(y) \tag{2}$$

Metal:

$$E[Q(z)] = E\left[Z(v)1_{Z(v) \geq z}\right] = E\left[1_{y_v \geq y}\phi_r(Y_v)\right] = \int_y \phi_r(u)g(u)du \tag{3}$$

where:

g and G are the standard Gaussian pdf and cdf

y is the gaussian cut-off related to z through $z = \phi_r(y)$

Reminders on the uniform conditioning in the univariate case

UC by panel grade (Rivoirard, 1994) aims at estimating the recoverable resources on a generic selection block V randomly located within a large block or panel V, conditioned on the sole panel grade, or for more generality, the panel grade estimate Z(V)*. Tonnage and metal at cut-off z are then:

$$[T_v(z)]^* = E\left[1_{Z(v) \geq z} | Z(V)^*\right] \tag{4}$$

$$[Q_v(z)]^* = E\left[Z(v)1_{Z(v) \geq z} | Z(V)^*\right] \tag{5}$$

The estimation of the metal at zero cut-off must then satisfy the relation: E[Z(v) | Z(V)*] = Z(V)*. It implies that the panel grade estimate Z(V)* has to be conditionally unbiased, ie E[Z(V) | Z(V)*] = Z(V)*.

The model also assumes that the Gaussian anamorphosis of Z(V)* is linked to that of Z(v):

$$Z(V)^* = E[\phi_r(Y_v) | Y_{V^*}] = \phi_{r\rho_{vV^*}}(Y_{V^*}) = \phi_s(Y_{V^*})$$

Hence the relationship:

$$S = r\rho_{vV^*} = r\ corl(Y_v, Y_{V^*}) \tag{6}$$

It is used to compute the correlation ('corl') between the block and the panel estimate:

$$corl(Y_v, Y_{V^*}) = \rho_{vV^*} = S/r$$

The ore tonnage and metal at cut-off $z = \phi_r(y)$ are then:

$$[T_V(z)]^* = E\left[1_{Z(v) \geq z} | Z(V)^*\right] = E\left[1_{Y_v \geq y} | Y_{V^*}\right] = 1 - G(a) \tag{7}$$

$$[Q_V(z)]^* = \int_a \phi_r\left(\rho_{vV^*} Y_{V^*} + \sqrt{1-(\rho_{vV^*})^2}\, u\right) g(u)du \text{ with } a = \frac{y - \rho_{vV^*} Y_{V^*}}{\sqrt{1-(\rho_{vV^*})^2}} \tag{8}$$

Uniform conditioning in the multivariate case

Multivariate uniform condition consists of estimating the recoverable resources of blocks v in panel V from the panel estimates ($Z_1(V)^*$, $Z_2(V)^*$, ...).

The problem is simplified by making the following assumptions (i denotes the index of a secondary variable 2,3, ...):

- $Z_1(v)$ is conditionally independent of $Z_i(V)^*$ given $Z_i(V)^*$, and so the UC estimates for the main variable correspond to the univariate case.

similarly, $Z_i(v)$ $Z_2(v)$ is conditionally independent of $Z_1(V)^*$ given $Z_i(V)^*$.

- $Z_1(v)$ and $Z_i(V)$ are conditionally independent of the other metal panel grades given ($Z_1(V)^*$, $Z_i(V)^*$)($Z_1(V)^*$, $Z_2(V)^*$). It follows that the multivariate case reduces to a bivariate case. In particular we have:

$$[Q_{iV}(z)]^* = E[Z_i(v) 1 z_1(v \geq z) | Z_1(V)^*, Z_i(V)^*]$$

The development of the equations makes practical computations achievable (Deraisme, Rivoirard and Carrasco, 2008).

The important point is that the multivariate model requires correlations between all variables and one main variable. The choice of that variable is then of prime importance particularly because the correlations between the secondary commodities are not directly modelled but are partly inferred through their respective relations with the main variable. In order to take into account the complete set of correlations, additional research would be required. Besides, it should be noted that the panel estimates must be calculated using co-kriging.

CASE STUDY

Geology

The case study is based on a porphyry copper gold deposit in Peru. The mineralisation is found in intrusive rocks within sedimentary rocks. Oxidation, weathering, leaching and subsequent secondary enrichment has led to the formation of four mineral domains with distinct different metallurgical behaviour. The top-most domain, the oxide domain, is characterised by the complete removal of copper mineralisation through the action of oxidation and leaching. Gold mineralisation within the oxide domain is characterised by some improvement in grade and is free milling due to the complete breakdown of primary sulfide minerals. All of the ore beneath the Oxide Domain comprises parts of the sulfide zone, which is separated into three domains on the basis of degree of oxidation and consequent change in sulfide mineralogical composition. The sulfide zone has three main domains, which from top to bottom are the mixed domain, the supergene domain and the hypogene domain. The supergene domain is an enriched copper blanket comprising chalcocite-covellite-chalcopyrite. The study presented in this paper was conducted in one of the hypogene domains (the annulus domain) which has a significant economic importance to the mine. The variables studied are AUTOT, CUTOT and NSR.

This most important economic domain (red colour in Figure 1) shows an annular morphology, which required specific processing of the variogram calculation and also at the kriging stage.

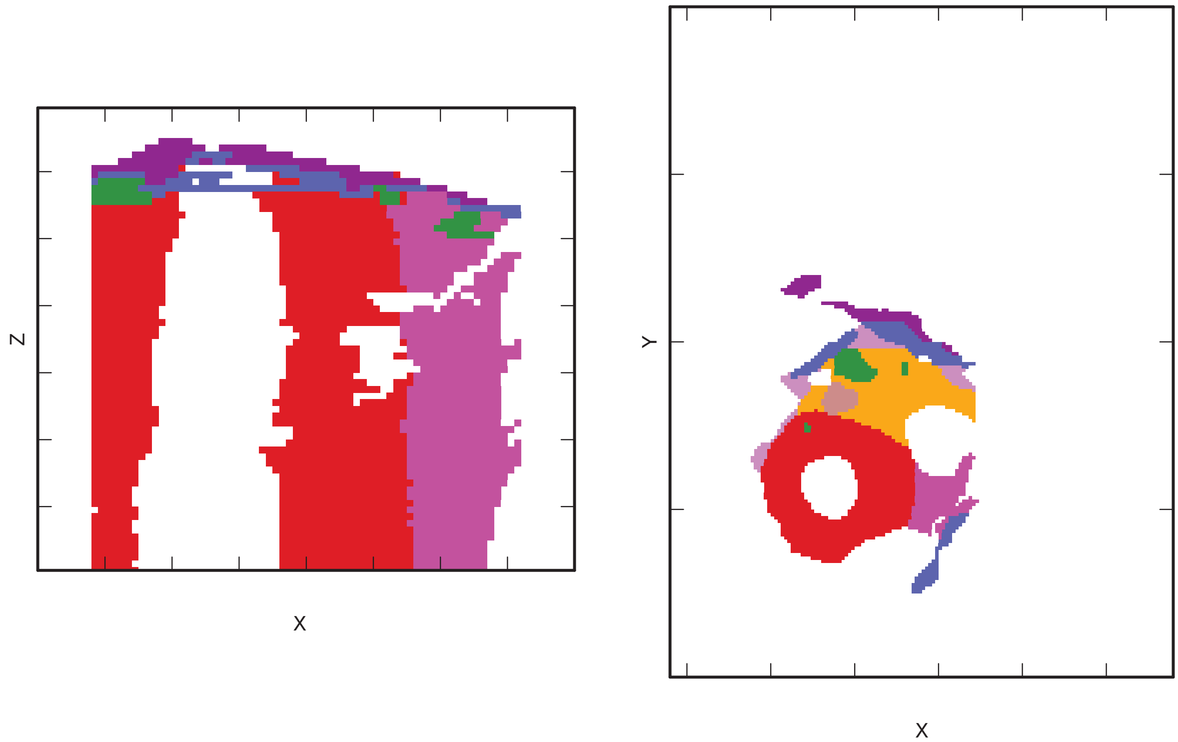

FIG 1 - Example of west east and plan sections with coloured estimation domains.

DATA ANALYSIS

The composited data on 2 m have been used to perform LMUC on 10 m × 10 m × 10 m SMU support from UC calculated on 50 m × 50 m × 10 m panels. Figure 2 provides the drill hole layout for the annulus domain.

The three variables have a positively skewed distribution (Figure 3) with coefficients of variation from 0.65 to 0.85. The correlations are highly significant (Table 1).

Declustering weights have been applied to calculate histograms and variograms. The experimental variograms are calculated separately in four sectors of the plane due to the curvilinear nature of the domain. For the directions in the horizontal plane the variograms of east/west are averaged with the variograms of north/south in the perpendicular directions. At the end two horizontal directional variograms have been kept (one representing the radial variability, the other the tangent variability) and one vertical variogram (Figure 4). A more rigorous variogram calculation not presented in this paper which takes the curvilinear co-ordinates directly into account without any averaging shows similar results.

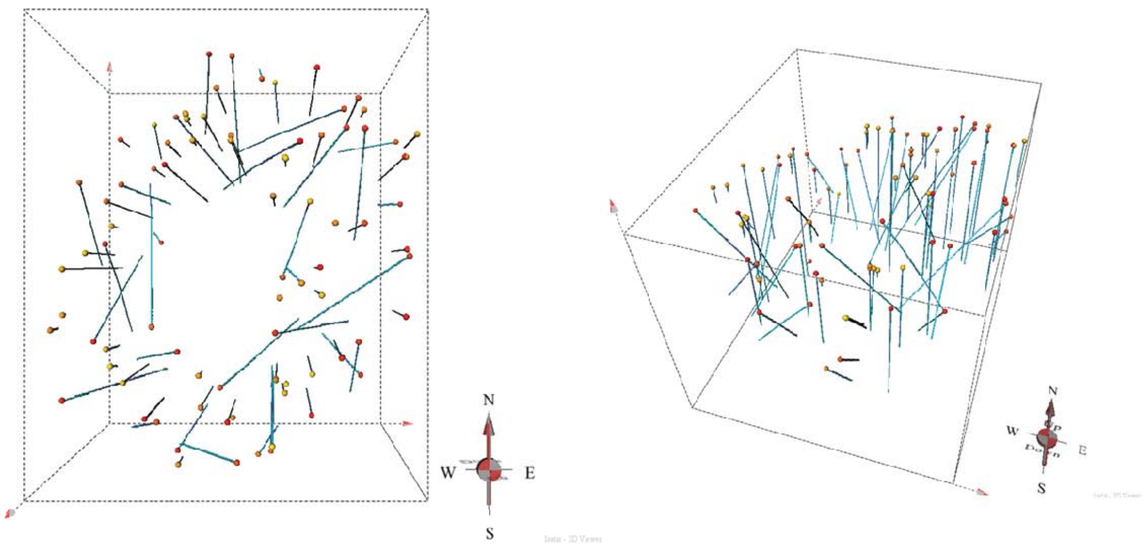

FIG 2 - Drill holes layout of the main domain displayed in horizontal projection and in perspective.

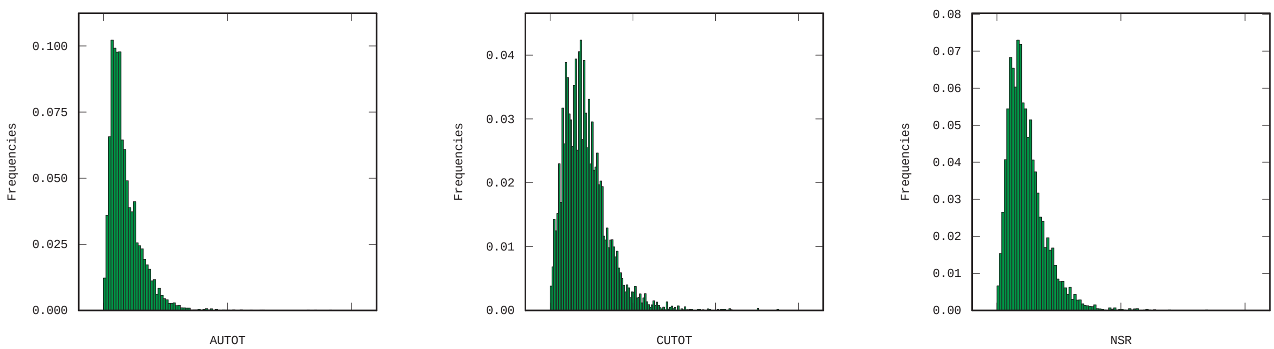

FIG 3 - Histograms of the 2 m composites for AUTOT, CUTOT, net smelter returns variables.

TABLE 1

Matrix of coefficients of correlation between three variables on 2 m composites.

	CUTOT	AUTOT	NSR
CUTOT		0.69	0.88
AUTOT	0.69		0.95
Net smelter returns (NSR)	0.88	0.95	

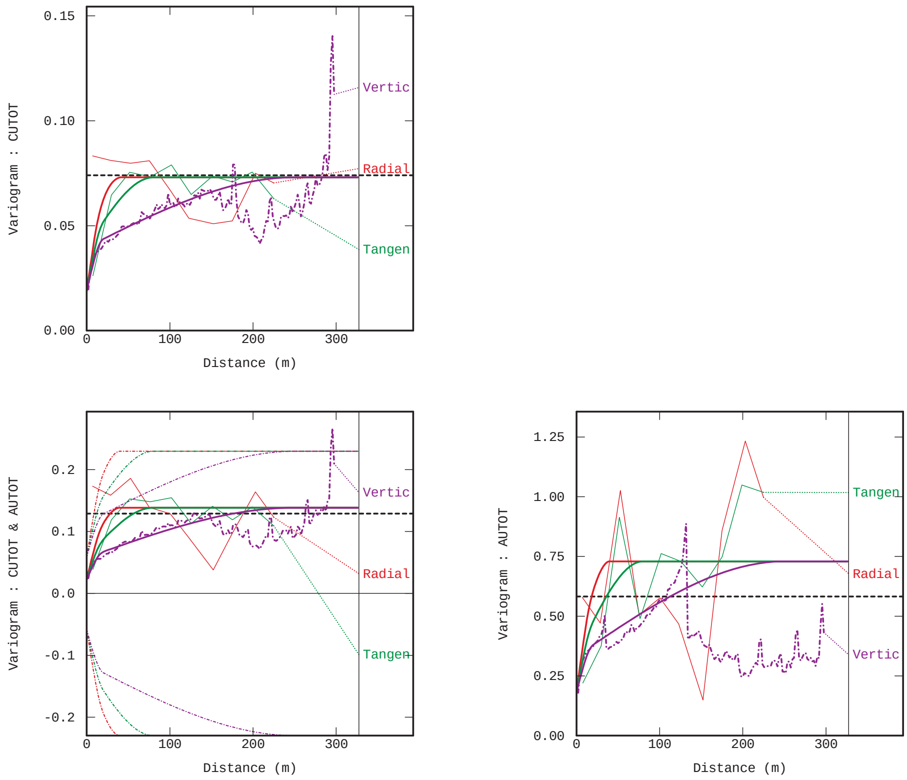

FIG 4 - Experimental and modelled variograms for CUTOT and AUTOT variables.

At the cokriging stages (for panels and SMUs) the orientation of the anisotropy of the variogram and of the neighbourhood is a parameter depending on the grid node location. This local parameter is then introduced to establish the kriging system.

Results

Change of support models

The distribution of 2 m composites has been modelled using a Gaussian anamorphosis function decomposed into Hermite polynomials. The change of support on SMUs is then achieved; the coefficients are calculated according to the selected main variable. The interpretation of these coefficients (Table 2) as coefficients of correlation between different variables in the Gaussian space shows that the correlations between block values are a bit higher than the correlations on composites.

TABLE 2

Change of support coefficients on selective mining unit support when the main variable is NSR.

	Net smelter returns (NSR)	CUTOT	AUTOT
Punctual variance (anamorphosis)	276.117	0.08	0.528
Variogram sill	270.45	0.076	0.536
Gamma(v,v)	128.191	0.045	0.212
Real block variance	147.926	0.035	0.316
Real block support correction (r)	0.7754	0.69	0.8285
Kriged block support correction (s)	0.7754	---	---
Kriged real block support correction	1	---	---
Main-secondary block support correction	---	0.8733	0.9804

Localised multivariate uniform conditioning

Figure 5 compares the recoverable SMU grade tonnage curves (GTC) based on three different techniques after applying NSR cut-offs, which are:

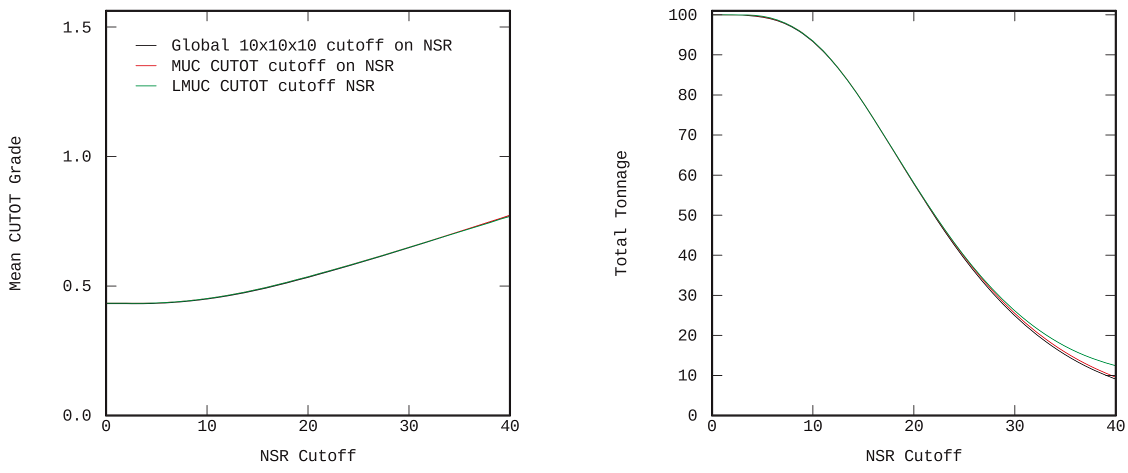

FIG 5 - Grade tonnage curves based on net smelter returns cut-offs for global change of support model, multivariate uniform conditioning and localised multivariate uniform condition.

1. GTC based on global change of support,
2. non-localised indirect GTC using MUC but on a basis of SMU functions within panels, and
3. LMUC based on localised SMUs.

The figures below show similar recoverable GTC results and provide a basis for using LMUC estimates for medium to long-term mine planning.

The main advantage of the LMUC approach is to derive a non-smoothed SMU grades with variability closer to the future production SMU block grades (see Figures 6 and 8).

The following is a summary of additional important results of the study:

1. The results further show that the grade variability of CUTOT when it is used as the main variable or a secondary variable is not preserved (see Table 3), which is logical since in the first case no information coming from other variables is used.
2. The study also demonstrates that the correlation between CUTOT and AUTOT shows similar results whether the main variable is CUTOT or NSR, though in the latter case, the correlation between AUTOT and CUTOT is not explicitly introduced (Table 4 and Figure 7).
3. Table 4 further shows that if the grades had been assigned independently by LUC carried out independently on both elements, the resulting correlation would have been poorly reproduced (with a lower correlation value of 0.53). In that case it would have been necessary to rearrange the grades assignment to increase the correlation to a realistic level.

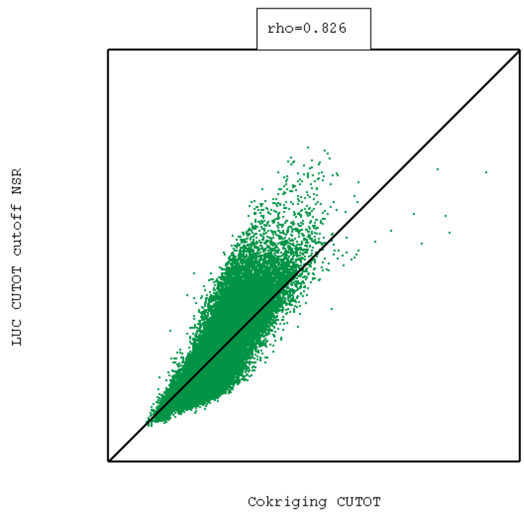

FIG 6 - Scatter diagram of the CUTOT grades assigned to selective mining units by localised multivariate uniform condition with net smelter returns cut-off and cokriged grades.

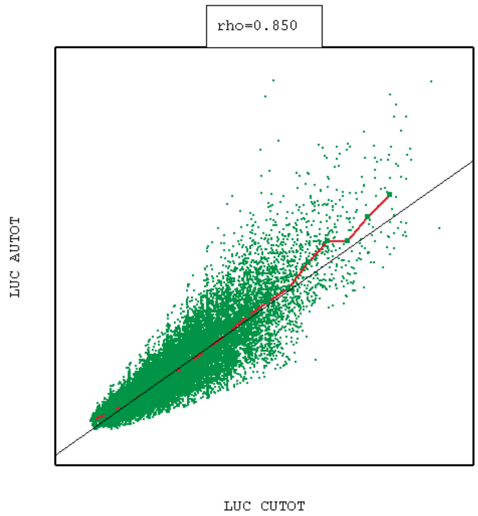

FIG 7 - Scatter diagram of AUTOT and CUTOT grades assigned by localised multivariate uniform condition with CUTOT as main variable. The black line is the regression line and the red broken line is the conditional expectation of AUTOT versus CUTOT.

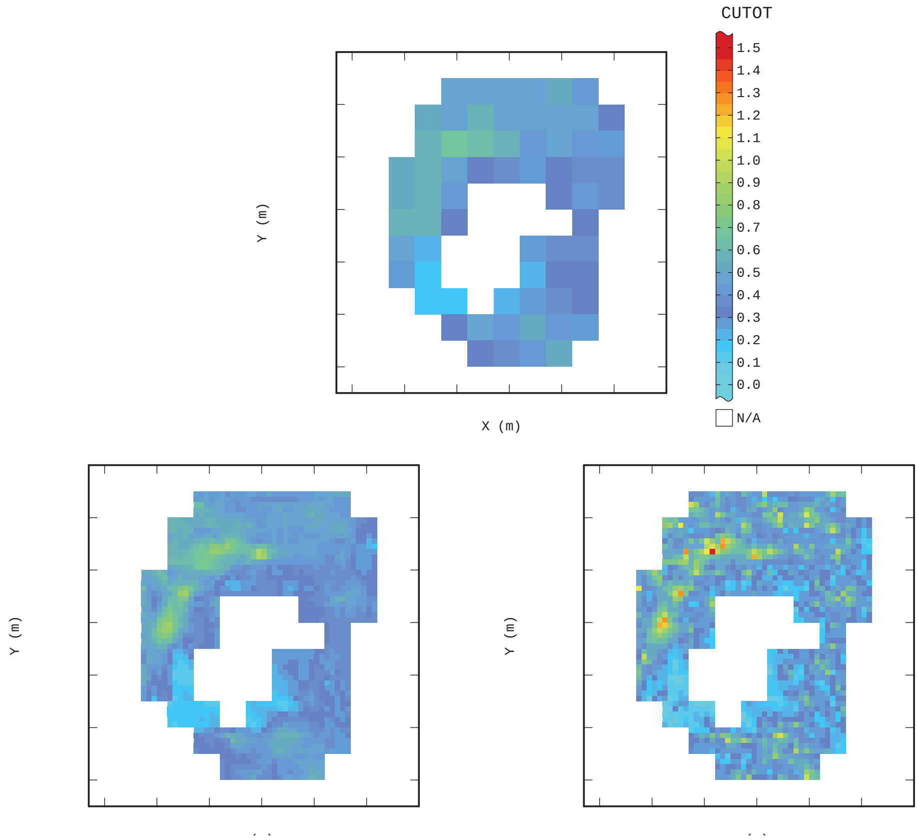

FIG 8 - Typical Example of CUTOT estimated grades on one bench, clockwise from the top: kriged panels, kriged selective mining units, selective mining units indirectly estimated by localised multivariate uniform condition.

TABLE 3

Matrix of coefficients of correlation (on the diagonal is given the variance for different SMU estimates on the SMU support).

Variable	Cokriging CUTOT	LUC CUTOT|Net smelter returns cut-off	LUC CUTOT|CUTOT cut-off
Cokriging CUTOT	0.015	0.8258	0.7851
LUC CUTOT|net smelter returns cutoff	0.8258	0.0298	0.8259
LUC CUTOT|CUTOT cutoff	0.7851	0.8259	0.0415

TABLE 4

Coefficient of correlation between the two element grades, AUTOT and CUTOT, of selective mining units for the different estimates.

Cokriging	0.68
Localised uniform conditioning independent for CUTOT and AUTOT	0.53
Localised multivariate uniform condition with CUTOT on CUTOT	0.85
Localised multivariate uniform condition with CUTOT on net smelter returns	0.84

Figure 8 demonstrates the advantage of the localisation method to be used for mine planning. The grades estimated by kriging or co-kriging of panels or SMUs are very smoothed when compared to that indirectly estimated using the LMUC technique. Any capital intensive project decision made on the basis of any of the smoothed estimates will have obvious misrepresentation of the economic value of the project or the operation, ie the average grade of the blocks above cut-off will be underestimated and the tonnage overestimated.

Extension of localised multivariate uniform condition to multi domains deposit

The LMUC algorithm may be applied on panels intersected by different domains with some care. Considering the SMUs as homogeneous (ie they are entirely in a single domain) two possibilities are offered:

- The tonnages and metal quantities of the panels are calculated from the contributions of the different domains proportionally to the domain volumes within the panel. Once all the domains have been processed, LMUC is achieved on the global tonnage and metals figures. The ranking is based on the estimated SMUs calculated with the specific parameters (variograms and neighbourhood) of the domains they belong to. If a panel is partially in waste, the complementary tonnage contributes to the metals by adding a tonnage with zero metal. The corresponding SMUs have then zero grades. In that case only the global tonnage and metals are kept, but it may happen that the tonnage and metals of one domain are not assigned to the SMUs of the same domain.
- Alternatively, after having calculated the tonnage and metals of one domain, representing a portion of the panel, these quantities are distributed to the SMUs of the same domain by the LMUC algorithm before processing the other domains present in the panel. The waste part of the panels has not been considered at all and the corresponding waste SMUs do not receive any grade. Thus, in this case as against the previous method, the metal of one domain is assigned to the SMUs of the same domain. The drawback is only when a panel is split into many domains. In that case it becomes difficult to assign grades corresponding to high cut-off grades, and the SMUs have smoother grade distribution than in the first case.

In practice, the difference between these two methods is marginal as it only concerns small domains whose tonnage is located into many panels. Figure 9 shows for one domain where most differences have been observed, ie for the different grade tonnage curves obtained using both procedures of achieving LMUC, globally or domain per domain. On all domains together both methods give at zero cut-off the same metals.

CONCLUSIONS

Gaussian models used for calculating recoverable resources provide consistent results in modelling the change of support and the information effect in the multivariate case. The UC method meets the goal of reproducing the correlation existing between the different grade elements at the panel scale. Using the grade tonnage curves thus obtained for each panel, the generalisation of the localisation methodology proposed on one or several elements is straightforward. It provides a practical advantage, in that no specific hypothesis on the correlation between the respective secondary elements is required though additional research would be necessary to incorporate these secondary correlations. Besides, there is no obstacle to the application of the same methodology to complex deposits divided into several domains. The important question that has still to be answered before applying that method is how to choose the main element on which the cut-off will be applied which consequently determines the ore tonnage above cut-off for the other secondary metals or elements. Additional research work would be required in this regard. Because of difficulties of modelling linear model of co-regionalisation for many variables, the necessity to split the variables into different

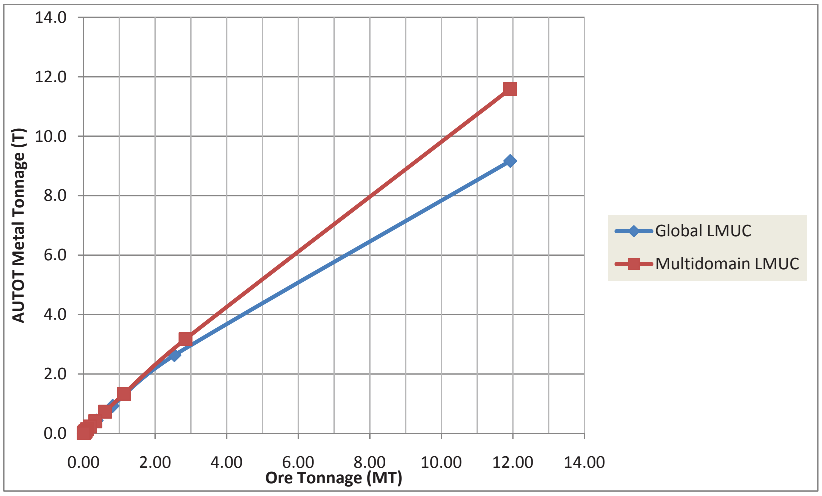

FIG 9 - Comparison on one domain of AUTOT metal versus ore tonnage calculated after cut-off on CUTOT from localised multivariate uniform condition selective mining unit grades assigned according to two methods: globally or specifically for each domain.

groups may appear as an obvious solution. The best combinations of variables depend on the correlation they have, but different solutions would have to be precisely evaluated and researched.

The study further shows that, the LMUC technique provides initial individual post-processed localised multivariate SMU recoverable estimates, to be used for feasibility or medium to long-term mine planning.

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