Recoverable resources estimation: Indicator Kriging or Uniform Conditioning?

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Introduction

Consideration of the mining method is an essential component of ore reserves evaluation. This is particularly true when the profitability of a project is conditioned by the ability to mine selectively.
Linear estimation methods such as Ordinary and Simple Kriging commonly fail to provide unbiased estimates of recovered ore and metal tonnage after cutoff, which means that a mining project can be exposed to undue risk.
This risk is significant when the selective mining are small with respect to the data spacing, resulting in over-smoothed estimates. Non linear estimation techniques are then necessary. Among these methods the most used ones are Indicator Kriging (IK), Multi-Indicator Kriging (MIK) and Uniform Conditioning (UC).

Indicator kriging methods

The estimation of ore and metal tonnage above a cut-off z in a panel V (of tonnage V_0) is derived from the indicators' estimates at the given cut-off:

$$T(z) = V_0 \times I^*_{KV,Z(x)>z}$$

$$Q(z) = V_0 \times E[Z(V)] \times I^*_{KV,Z(x)>z}$$

The kriged estimates of the indicators for a panel will then be interpreted as the probability of a sample located within the panel to be above the cut-off, or the proportion of samples above the cut-off within this same panel. The kriging uses the variogram of each indicator (MIK). But doing so it classically produce some inconsistencies when comparing the different cut-offs: hence, decreasing tonnages for two increasing cut-offs. A way to prevent these problems would be to perform an indicator co-kriging, which will take all the correlations between different indicators into account. But it is quite heavy and somehow difficult to achieve, particularly because the required multivariate variogram model has to follow the constraints of the linear model of co-regionalization, difficult to fulfil for a large number of cut-offs..

An alternative and practical solution may be the kriging of all indicators with the **same** variogram. Besides the advantage to not require a specific variographic analysis for each cut-off, the number of inconsistencies is lower than with MIK. The limitation is that it applies only on specific spatial distribution of grades, described by a particular model called mosaic model (J.Vann, 2000). The question of which cut-off variogram is to choose also needs particular attention, even if the median cut-off is often chosen.

Moreover the different variants of indicator kriging consider so far that the decision between ore and waste is made upon a point support (equivalent to the sample size). In reality, the selectivity is highly dependent on the smu size support. . If we consider the panel V contains N smus v_i, then the ore tonnage's expression becomes:

$$T(z) = V_0 \times \frac{1}{N} \sum_{i=1}^{N} I^*_{K,Z(v_i)>z}$$

As the available data are not dense enough to estimate each smu individually, we have to take the change of support into account. To complete this, the tonnages calculated by indicator kriging are transformed with the assumption of punctual selection . Generally simple and somehow arbitrary hypotheses, based on the reduction of variance between samples and smus, are assumed..

Uniform Conditioning

Geostatistics provide some powerful methods to achieve change of support, i.e calculate the block grades distribution from the sample grades distribution. The global grade-tonnage curves can then be calculated for any smu support. Uniform Conditioning aims to condition the local grade-tonnage curves for each panel V. The so-called discrete gaussian model (Rivoirard J. 1994) is then applied, basically assuming, first, that the smus constitute a partition of the panels, and second, the gaussian values (after normal score transform) for smus and panels have a bivariate normal distribution, only depending on change of support coefficients.

If we name the raw grades Z and the "gaussian grades" Y , obtained from the normal score transform, we have the following relationship: Z=ϕ(Y), where $\phi$is an anamorphosis function. .Applying the anamorphosis to the different supports (x for samples, v for smus and V for panels), we get the following relationships:

$$Z(x) = \phi(Y(x)) \Rightarrow Z(v) = \phi_r(Y_v) \Rightarrow Z(V) = \phi_{r'}(Y_V)$$

The coefficients r and r' are called change of support coefficients and express the decreasing dispersion variance from points to smus, and from smus to panels.

Applying that model we can get the panel ore tonnage when the cut-off z is applied to the smu support:

$$T(z) = E\left[I_{Z(v)>z} \middle| Z(V)\right] = 1 - G\left(\frac{\phi^{-1}(z) - RY_V}{\sqrt{1-R^2}}\right)$$ with R=r'/r, G being the gaussian cumulative density function.

Note this formula only depends on the anamorphosis function and the change of support coefficients for smus and panels. The conditioning of the panel grades is achieved by means of its proper gaussian transforms. As the panel grade is unknown, we usually replace it by its kriged value. In this case, the change of support coefficient r' is calculated in order to match the kriged panel variance, instead of the actual panel variance.

The advantages of the uniform conditioning is that it is a rather straight forward method, based on ordinary kriging and hence not requiring strict stationarity.

Besides, we can incorporate the **information effect** to the estimation of the grade tonnage curves: during the production stage, the actual grades are recovered and may then be taken into account so the decision between ore and waste is made upon more accurate estimates of the smus. Therefore we can anticipate future decisions before obtaining the production blast-holes results, because only the kriging variance of these smus' final estimates is necessary. Figure 1represents the information effect in terms of misclassification (rich ore sent to waste, in red on the figure, and poor ore added to recovered ore tonnage, in blue).

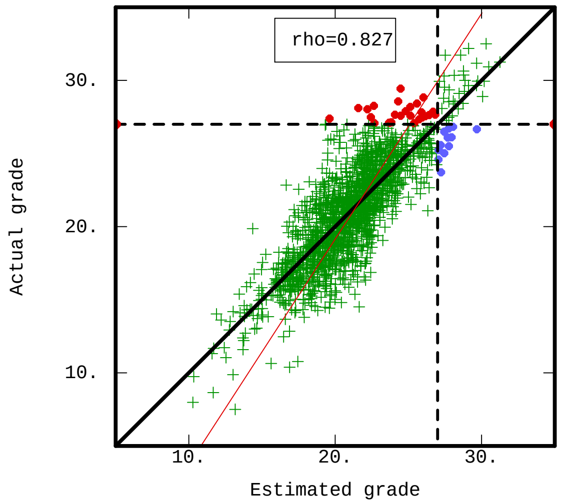

Figure 1: Scatter diagram of actual grades vs. estimated grades and resulting misclassification.

Case study

Different methods have been tested on the Walker Lake data set (Isaaks E. and Srivastava M. 1989), quite well known in the mining industry. The sampling is preferentially densified in the high-grade areas.

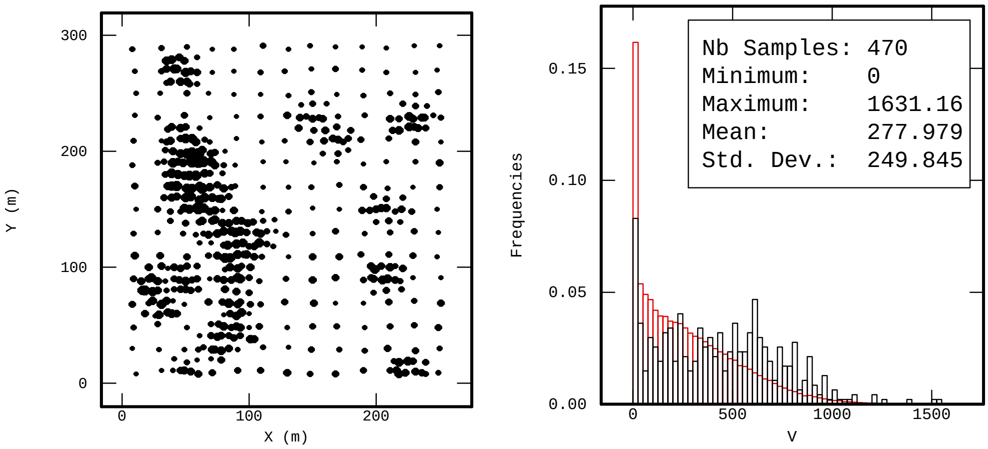

Figure 2: Base map of Walker lake sample set (the symbol size is proportional to the grade) and histogram before (in black) and after declustering (in red).

The variograms of indicators from 0 to 1000ppm have been calculated by steps of 50 ppm. They are less and less structured as the cutoff departs from the median cutoff (about 450ppm) increasing the nugget effect. The indicators have been kriged in panels 20mx20m using either the same median indicator variogram, or the specific variogram of each specific cutoff. For both methods an affine correction of support has been applied, meaning that the distribution keeps the same shape and is not deskewed for the smu support of 5mx5m.

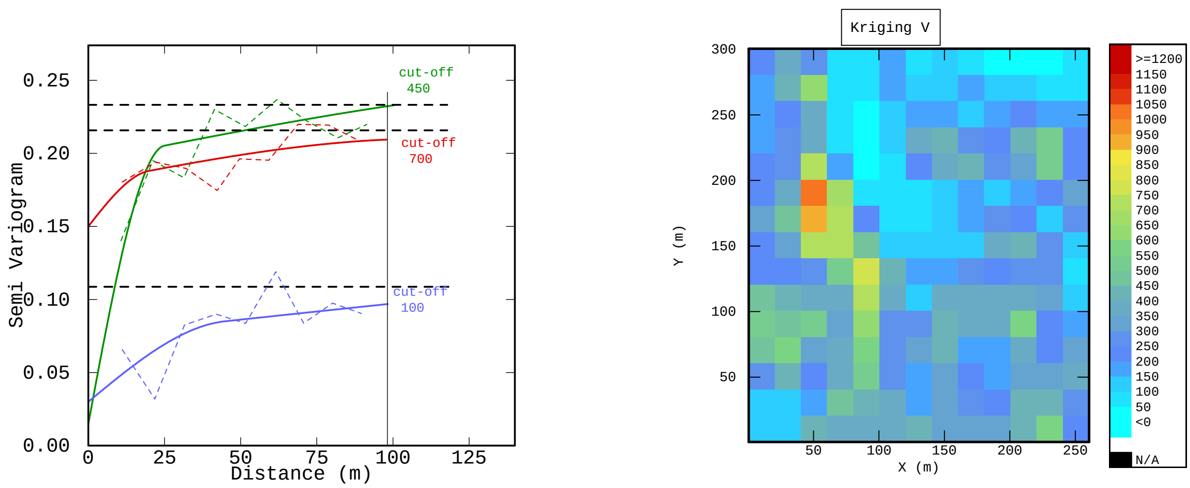

Figure 3: Variograms for a series of 3 cutoffs and kriging of the average grade.

The average grade of the panels has also been kriged, and used to condition the distribution of smus grade for the uniform conditioning method. A change of support model based on the discrete gaussian model has been applied for transforming the distributions (sample to smu and to the kriged panels). The notable advantage, compared to affine correction, is that it considers the symmetrization of the grade distribution for large panels, with respect to the long range of the variograms. The grade tonnage curves obtained with the three methods (IK, MIK, UC) have been compared to the supposed reality, as it can be obtained from the exhaustive Walker Lake data set

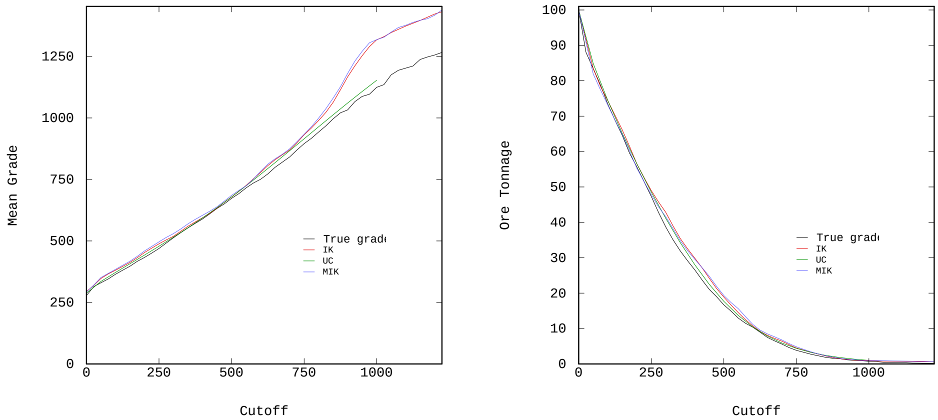

Figure 4: Grade tonnage curves comparing IK, MIK and UC to the “reality”.

All 3 methods appear to work well and provide similar recoverable resources estimates for a large range of cutoffs, the methods being ranked as MIK, IK and UC from the furthest from reality to the closest. The conventional profit is then calculated from the ore tonnage T, metal quantity Q and cutoff z as: P= Q-z*T. It is a simplified profit which behaves similarly to the actual profit. It enables to better interpret the methods, particularly it appears that the profit is overestimated whatever the used method, but Uniform Conditioning has a half-reduced bias compared to MIK.

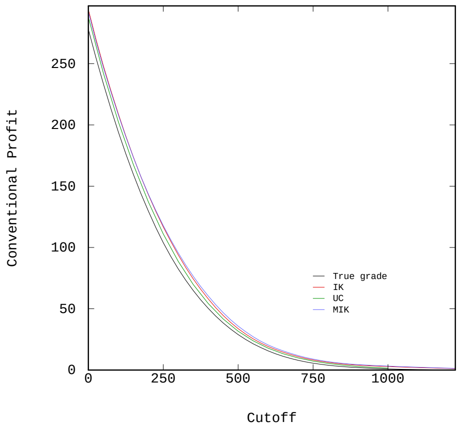

Figure 5: Conventional profit from IK, MIK, UC estimates compared to the “ideal” case.

Conclusion

The geostatistical non linear methods can be classified in 2 categories: methods based on indicator kriging with a simplified change of support correction, and the methods based on a model of the grade distribution with a consistent change of support model. The applicability of these methods has been proven in many cases, but the choice between them strongly depends on the spatial grade distribution and the small scale variability .

References

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