On the use of non-linear geostatistical techniques for recoverable reserves estimation: A practical case study

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Abstract

Consideration of the mining method is an essential component of ore reserves evaluation. This is particularly true where the profitability of a project is conditioned by the ability to mine selectively.

Linear estimation methods such as Ordinary and Simple Kriging commonly fail to provide unbiased estimates of recovered ore tonnage and metal content which means that a mining project can be exposed to undue risk. This risk is significant when the selective mining are small with respect to the data spacing.

Non linear estimation, such as the Gaussian Disjunctive Kriging technique provide a mean of calculating unbiased estimates of ore and meta content over any cut-off range and mining unit size combination. The disadvantage of this methods is the requirement of an assumption of strict stationarity. When such an assumption can not be reasonably made, alternative simpler non linear methods can be employed such as Uniform Conditioning and ore and metal Service Variables.

The application of these estimation methods to a porphyry type deposit is described. A discussion of the results from a practical point of view is also given.

Introduction

The objective of this paper is to demonstrate how recoverable reserve estimates vary according to the selection of estimation method. The applicability of three non linear estimation methods as applied to a porphyry copper deposit is discussed with reference to both the theoretical assumptions and the geological characteristics of this particular deposit. The relative performance of each method is assessed by comparing grade-tonnage curves and metal content estimates above cut-off thresholds. Additionally, conditional simulations are generated to determine which method has the best absolute performance and what practical recommendations can be made for the application of these techniques.

All of the statistical and geostatistical calculations and graphical output generated for this case study was made using the Isatis (trade mark of TRANSVALOR) geostatistical software system.

Presentation of the data

Copper mineralisation is located within a stockwork zone which is structurally controlled by major step-faults. The intensity of mineralisation is generally related to the degree of stockworking; at the deposit core where the host rock is most extensively stockworked, copper grades are highest. A decline in the intensity of stockworking away from this central zone is accompanied with a gradual decline in copper grades.

The deposit has been extensively explored from surface diamond drilling and underground exploration drives, cross-cuts and raises. The sample database consists of a combination of diamond drillhole and channel samples. Statistical analysis confirmed that the support of these different sample types is compatible.

For the purposes of this case study, 5m composites over only one exploration level are considered which gives a total of 1256 composite values. The map given in Figure 1 shows that the location of composites is dictated by the spacing and orientation of the underground development.

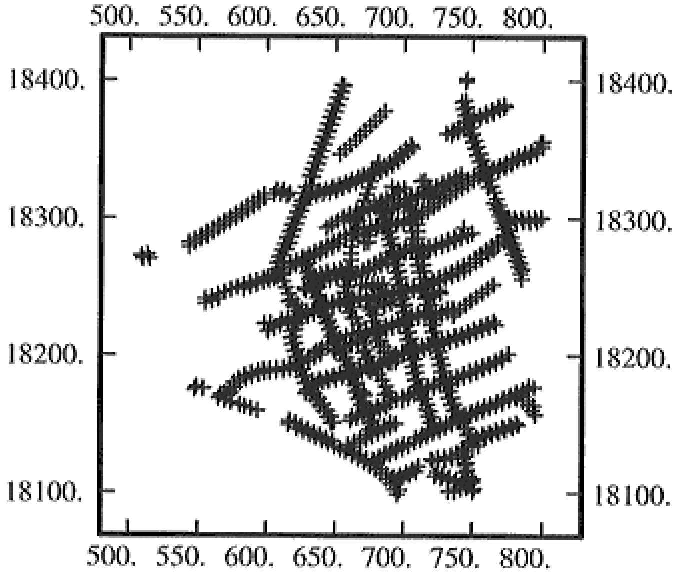

Figure 1

The data density is such that linear kriging into small blocks which approximate the selective mining unit (SMU) size may be appropriate. This approach would negate the need to use more sophisticated methods because no further support correction would be required. However, the copper grade distribution is highly skewed (Figure 2) with a coefficient of variation of greater than 2. This characteristic suggests that the use of linear kriging methods for the estimation of recoverable reserves is inadvisable because estimates are unlikely to be conditionally unbiased. The use of non linear methods is therefore a more suitable option.

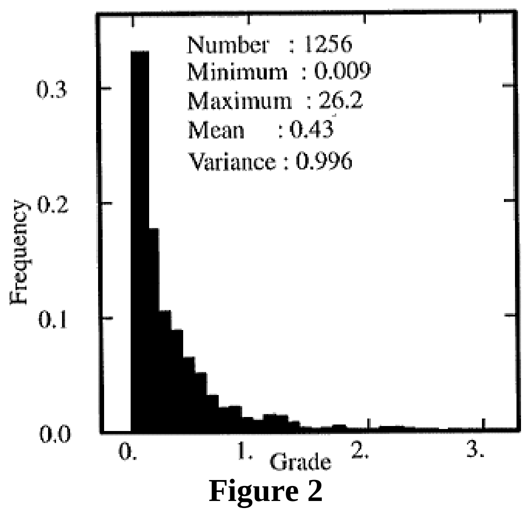

Figure 2

Declustering of the composite data was found to have little effect on the mean grade and variance of the sample distribution.

The existence of grade trends was investigated to determine what directions are important when calculating directional experimental variograms. Also the results of this analysis were used to assess if stationary conditions exist over the deposit. This analysis was performed by calculating a moving average within a 20m x 20m window. A plot of the results is presented in Figure 3.

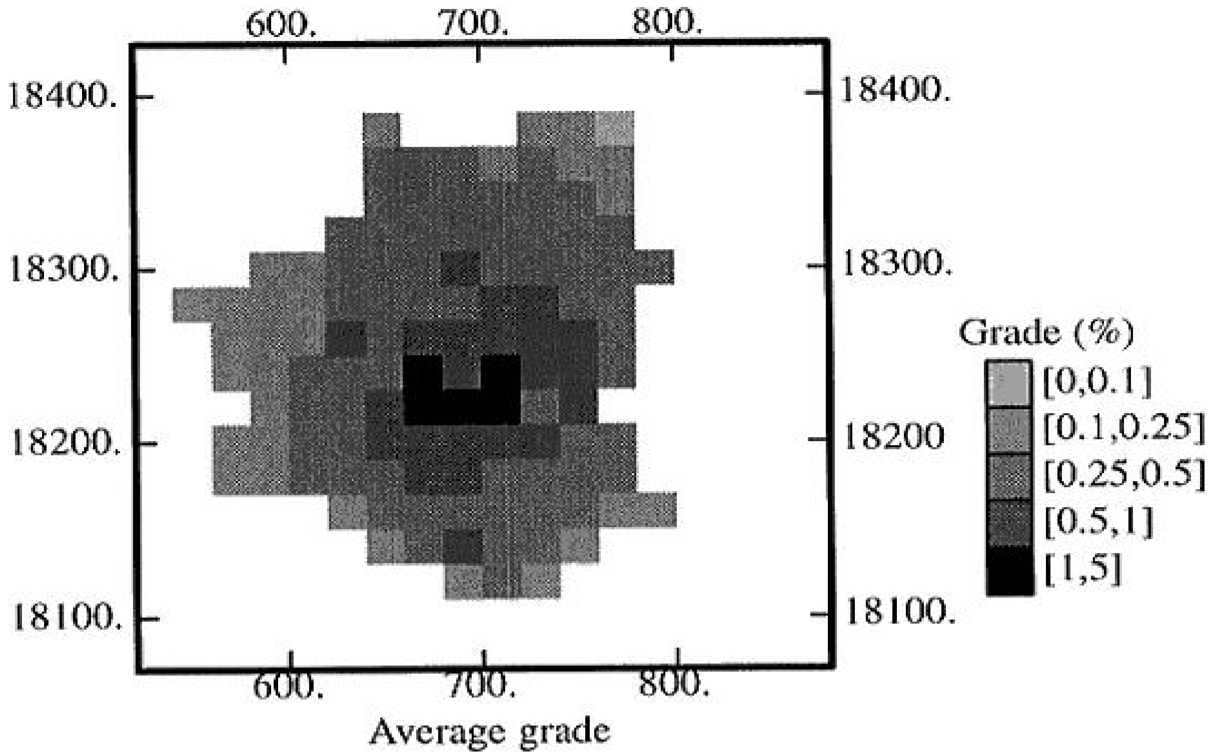

Figure 3

These results clearly confirm the geological observation that copper grades decrease in a roughly regular manner from the high grade central core of the deposit. There is a weak suggestion of greater grade continuity along the NE - SW axis. The change of the mean (and variance implied from the proportional effect) across the deposit indicates that stationary conditions do not exist.

Geostatistical analysis of data

The geostatistical analysis presented in this section firstly describes how the spatial correlation of the copper grade is characterised through the fitting of a variogram model, and secondly, the use of the discrete gaussian model which provides a means of deriving the distribution of grades on any support. This model is necessary for the application of the three non linear recoverable reserve estimation methods; Disjunctive Kriging, Uniform Conditioning and Service Variables.

Experimental Variogram Calculation and Modelling

Directional experimental variograms were calculated along the three primary orientations of the underground exploration drives where the maximum number of sample pairs are found; 3450, 0700 and 1150. The direction 0700 lies approximately along the direction of greatest grade continuity as indicated in the grade trend analysis.

Given the highly skewed grade distribution, the spatial structure is likely to be difficult to interpret. The experimental plot (Figure 4) shows, however, that a structure can be easily interpreted. This plot confirms the anisotropic nature of the copper mineralisation with the major axis of continuity along the 0700 direction. The sill along azimuth 3450 is interpreted to be higher and a model has been fitted which includes both geometric and zonal anisotropic structures.

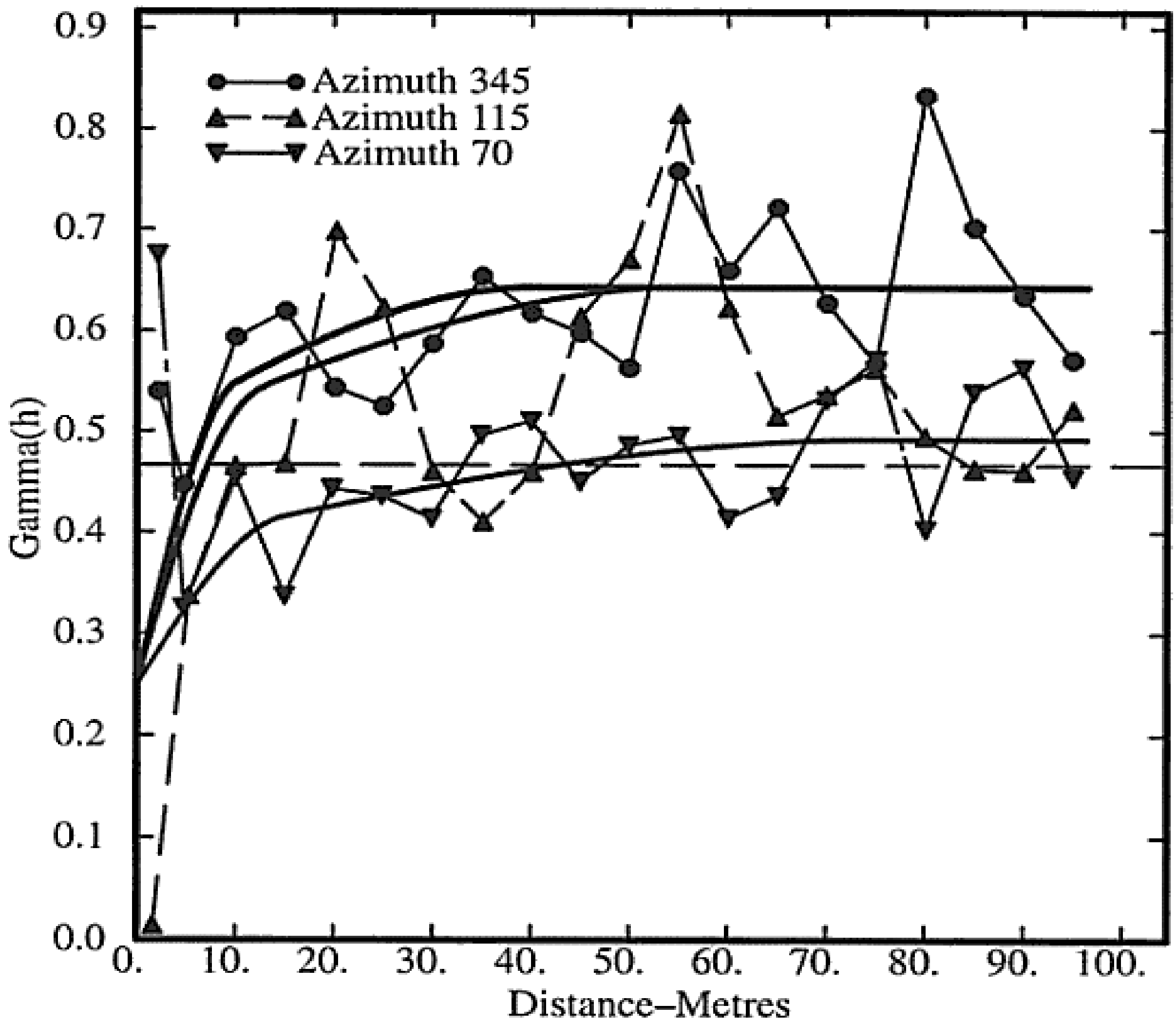

Figure 4

The fitted model expression is as follows:

$$\gamma(h) = 0.25 + 0.13 \times Sph(15,10) + 0.11 \times Sph(75,10) + 0.15 \times Sph(,40)$$

The ranges of the spherical models are expressed for the two directions 0700 and 3400. Where the range is not defined (last spherical structure), then it corresponds to a zonal structure.

Change of Support Model

The objective of using a change of support model is to predict the distribution of grades at a volume equivalent to the SMU size, given the grade distribution of composite (point) sized volumes.

For this application, the discrete gaussian model was used which requires a normal scores transform of the composite data. Given the hypothesis that the transformed data conform to a bivariate normal distribution, the grade distribution of the SMU sized volumes can be derived by calculating the change of support coefficient r which is linked to the difference in dispersion variance between the point and SMU sized volumes. The dispersion variance itself can be calculated from the point variogram model regularized on to the SMU support.

The normal scores transformation of the composite data was achieved by fitting a Hermite polynomial function to the anamorphosis function linking raw (Z) and gaussian (Y) values:

$$Z = \Phi(Y) = \sum_{n=0}^{N} \Phi_n H_n(Y)$$

Difficulty can sometimes be experienced in the †tailsË of the distribution where extreme data values can make the function unstable. The quality of the fit achieved for this application was checked by comparing grade and tonnage curves for the original composite data (experimental) against that predicted from the discrete gaussian model at a point support (point model). This comparison together with the predicted curves on a SMU support, assuming a selection size of 2m x 2m, are given in Figure 5.

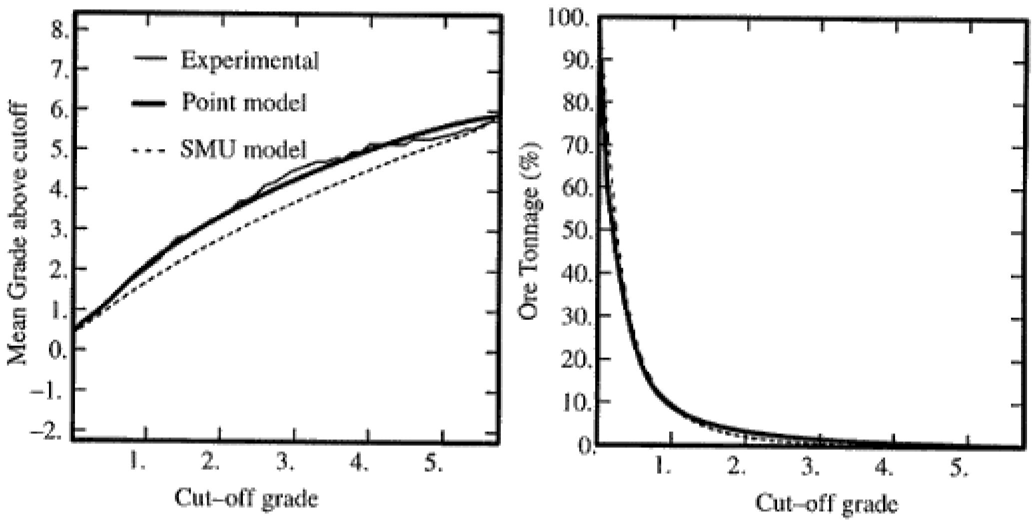

Figure 5

This method is known as a global change of support, whereas the non-linear methods provide a local change of support. These results show that the theoretical model satisfactorily reproduces the experimental curves.

Linear Kriging

Kriged estimates for copper grade were generated using Ordinary Kriging (OK) for two panel sizes; 20m x 20m panels; 2m x 2m panels.

Grade estimates were generated for the larger panel size because these values are required for the Uniform Conditioning method.

The objective of generating estimates for the smaller panel size, which is equivalent to the SMU size, is to illustrate that as a result of the information effect, kriging directly into small panels produces a grade distribution that is too smooth. This will be demonstrated by comparing the OK grade-tonnage curves with those generated using non-linear methods.

The OK estimates for both panel sizes were generated using the modelled variogram parameters described previously. Extrapolation beyond the data limits was controlled by using a moving neighbourhood with a search of 15 angular sectors and a maximum of 3 empty sectors. The results of these kriged estimates are illustrated in Figure 6.

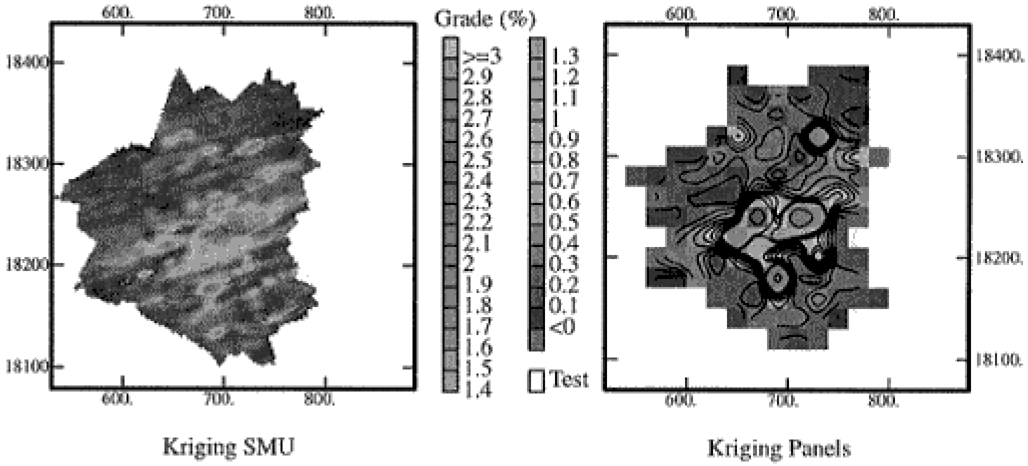

Figure 6

Estimation of local recoverable reserves

This section describes how local recoverable reserve estimates were developed using three different non-linear geostatistical techniques. Relative comparisons of the results of the different methods are also considered.

Disjunctive Kriging

The Disjuntive Kriging (DK) method independently kriges each of the terms in the Hermite polynomial function using the variogram of the gaussian transformed values on the SMU support.

The discrete gaussian model is used to provide a simple relationship between covariances of raw and gaussian values, respectively C(h) and (h):

$$C(h) = \sum_{n=1}^{N} \Phi_n^2 \rho^n (h)$$

Figure 7 shows the variogram model fitted to the experimental variogram of the gaussian values on the SMU support. This plot illustrates how the anisotropy has been preserved but the nugget effect has disappeared due to the increase from point to SMU support.

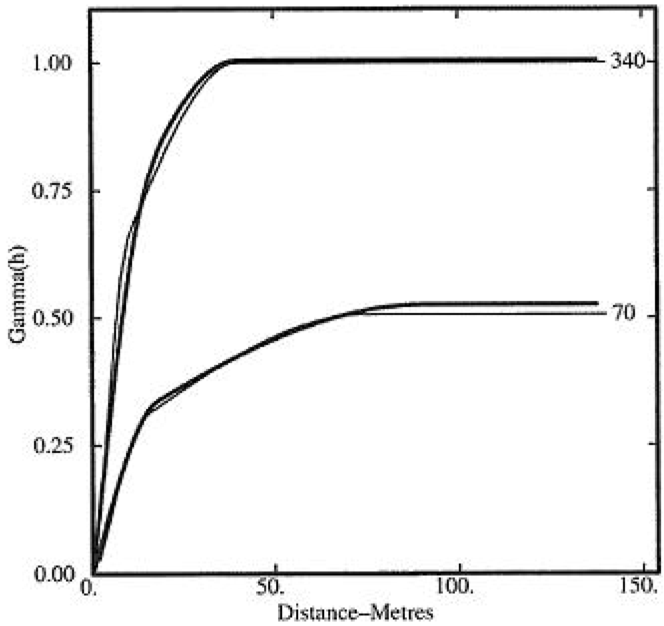

Figure 7

Grade-tonnage curves were computed from the DK estimates for the cut-off range between 0 and 1% Cu. Figure 8 illustrates how these results compare with those predicted from the global distribution of block grades, derived from the global change of support as described previously (see Figure 5).

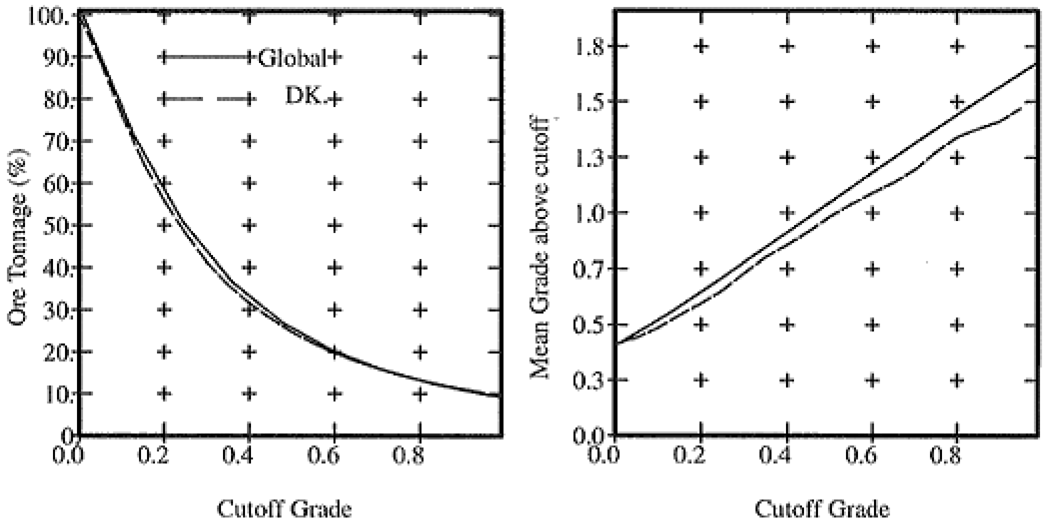

Figure 8

These results show that the tonnage curves for the local DK and the global anamorphosis methods match quite well. The grade curves, however, show a significant difference with the DK estimates lower than the global mean grade. This difference is likely to be due to preferential sampling in the higher grade areas which will result in the global method over-estimating grade above cut-off because high grade material is over-represented in the global grade distribution.

Uniform Conditioning

The second recoverable reserve estimation method considered in this case study is Uniform Conditioning (UC). This method has several practical advantages over the DK method described previously; firstly it is more robust than DK because it does not require strict stationarity assumptions, secondly the implementation of the method is simpler than DK and thirdly it is conceptually an easier technique to understand.

The first step in applying this method is to generate OK panel estimates for copper. This was described previously where of 20m x 20m panel estimates were generated over the level. The ore tonnage and grade in each panel at the SMU support found by applying the block anamorphosis function to find the local grade distribution.

Comparison of OK, DK and UC estimates

Figure 9 shows a comparison of the grade and tonnage curves for OK, DK and UC recoverable reserve estimates calculated over the grade interval of 0 to 1% Cu.

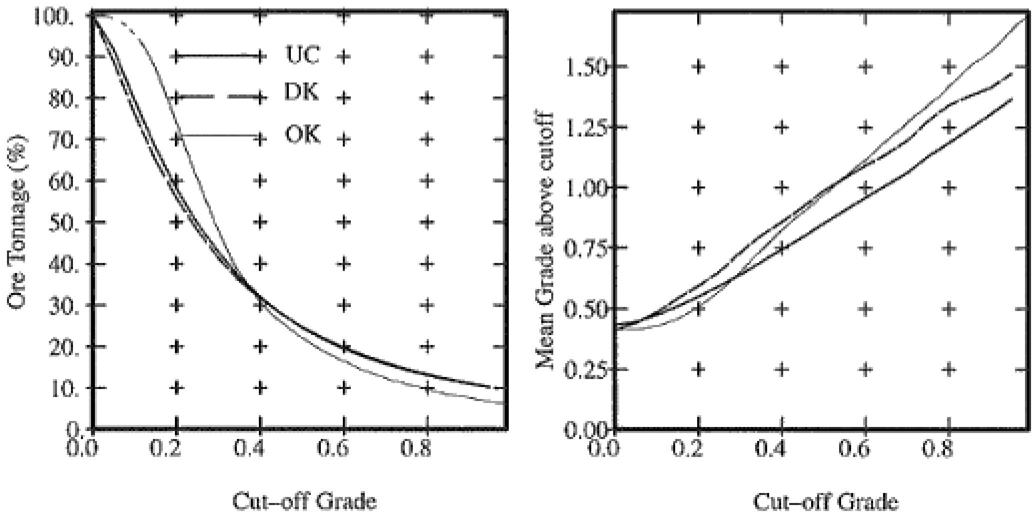

Figure 9

These results show that, as expected, the ore tonnage estimates calculated from OK is biased: under-estimation when the cut-off is below the mean grade, over-estimation otherwise. The difference in the shape of the curves illustrates how the OK estimates are smoothed relative to the other estimates which better represent the grade distribution of SMU sized panels.

The ore tonnage estimated by DK and UC match closely, whereas the DK method consistently predicts a higher grade. A possible reason for this difference is that the departure from the strict stationarity assumption effects the results of the DK estimates. This possibility was checked by investigating what weight was assigned to the mean value when kriging the Hermite polynomial terms; the larger the weight the more reliant the estimate is upon the stationarity assumption. It was found that the weight assigned to the first term was only between 5 to 20% which means that the source of the discrepancy is unlikely to be due to departures from this assumption.

The source of the discrepancy lies with the DK estimates, since the UC results compare more closely with other techniques, as demonstrated below. It is most likely that instabilities in the DK metal tonnage estimates occur due to restricted range of gaussian values within the neighbourhood search. This assertion is supported by observed discrepancies between the DK and UC metal tonnage estimates.

Service Variables

The third recoverable reserve estimation method considered in this case study is Service Variables (SV). The application of this method involves changing the support of the composite data to SMU size at the data point locations through the use of the change of support model. The proportion of tonnage and metal proportion above a cut-off threshold calculated at each data location is then used for kriging these values into the 20m x 20m panels.

Unlike the DK and UC methods, the method does not provide grade-tonnage curves for the full range of cut-off thresholds, but applies a single cut-off which is selected during the first step of the estimation process.

Comparison of DK and UC with SV estimates

For the purposes of this case study, the selected cut-off threshold for generating the SV estimates was 0.45% Cu which is approximately equivalent to the median copper grade. Figure 10 compares panel estimates for mean grade above 0.45% Cu for SV versus DK and SV versus UC estimates

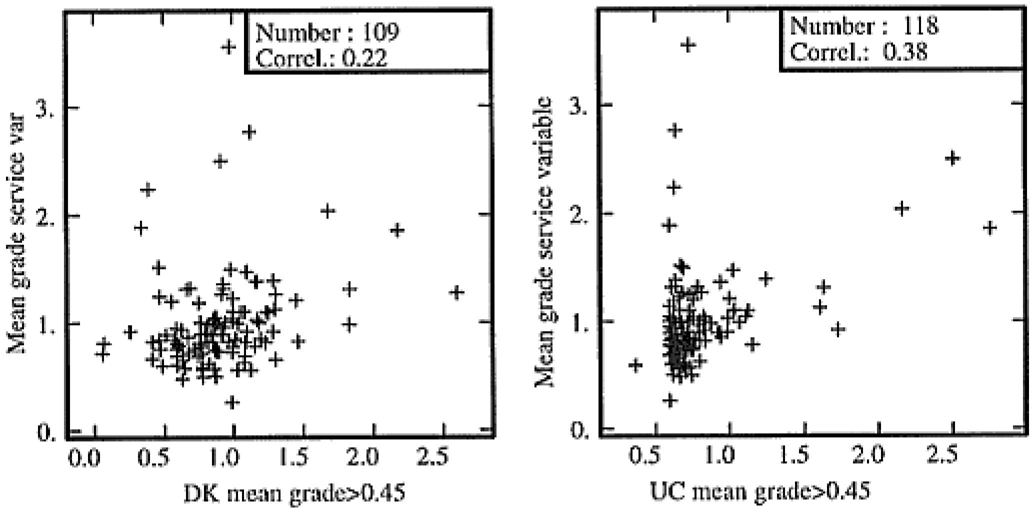

Figure 10

These plots show that the UC and SV values are better correlated than the DK and SV values. The scatter diagram of the DK and SV values show that the relationship between the two estimates is weak, whereas, with the UC and SV plot shows a closer relationship. This result supports the notion that some instabilities occurred when generating the DK estimates.

In terms of the practical application, the UC technique is simpler to apply than the DK method and it offers greater flexibility than the SV method because the entire grade-tonnage curve can be estimated rather values above a single threshold.

Comparison of recoverable reserve estimates and simulated values

The analysis presented above describes the relative performance of the recoverable reserve estimation methods. The absolute performance of these methods could be best tested against mine production data. However, this information is not available for this deposit because the mine is yet to be commissioned. An alternative solution is to conditionally simulate the copper grade values on a dense grid and then to average these realisations into SMU sized panels from which the grade and tonnage curves could be calculated. The advantage of this method over the use of estimation techniques is that there is no need to use a change of support model.

A total of 50 conditional simulations on a 0.66m mesh were generated using the Turning Bands method. The SMU grades were simulated by averaging 9 realisations within each 2m x 2m panel. A distribution of simulated grades was then generated for each SMU panel from which the expected tonnage and grade above the 0.45% Cu cut-off grade was calculated. Figure 11 illustrates how DK, UC and SV estimates compare to the simulated values.

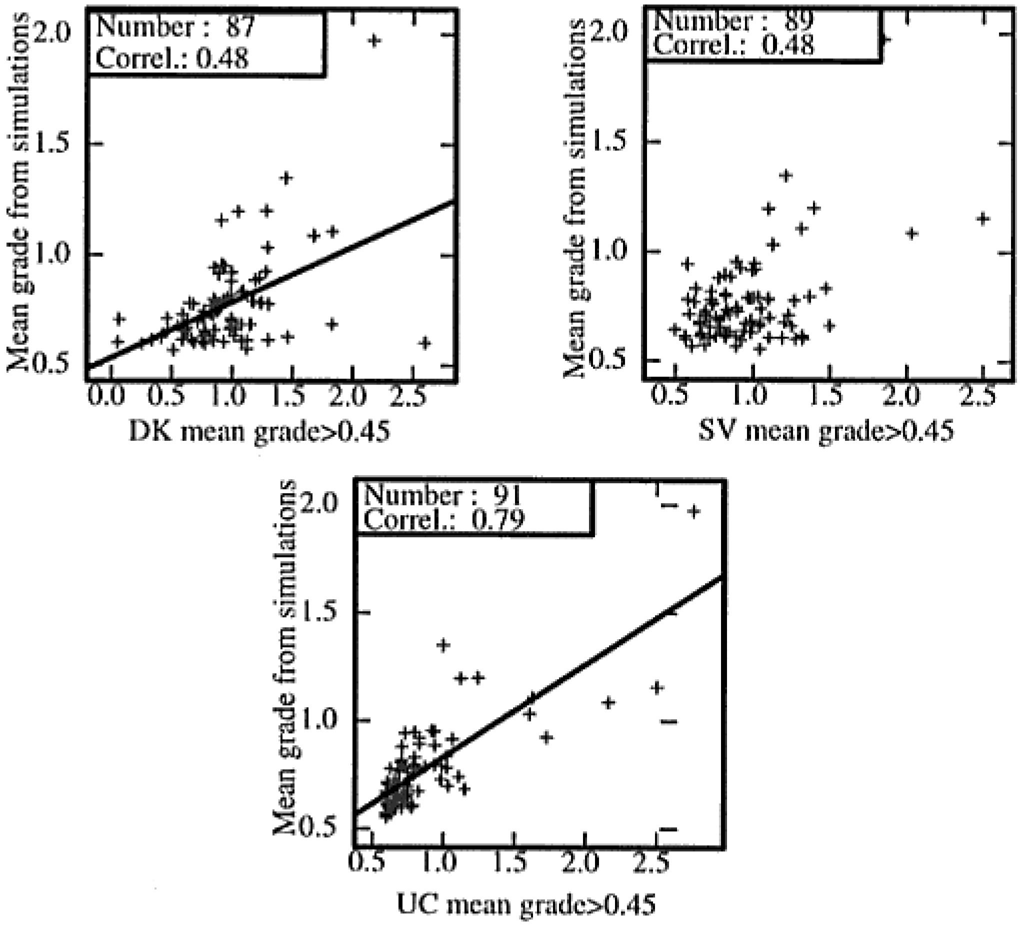

Figure 11

These results show that for this case study, the DK method performs least well against the simulated values. Even if the coefficient of correlation between the simulated values and SV estimates is the same as with DK, the shape of the scatter diagram is more satisfactory. The correlation between the UC estimates and simulated values is strongly suggesting that this method is suited to this application

Conclusions

This paper illustrates how different non-linear geostatistical methods behave from a practical perspective. The results presented in this case study show that Disjunctive Kriging, Uniform Conditioning and the Service Variables methods give almost the same results for ore tonnage above cut-off. These estimates also fit the tonnage estimates derived from the global change of support.

The results for the mean grade or metal content above cut-off grade show that the agreement between the methods is not as good. In particular, the DK method consistently over-predicts grade and metal content relative to the UC method. This discrepancy is thought to be due to the influence of preferential sampling, instability in the DK metal estimates and possibly departures from the strict stationarity assumption.

The comparison between conditionally simulated SMU grades and the three techniques confirmed that the performance of DK method for grade estimation is worst and the UC method is best. These results also confirmed that the hypothesis of permanence of the distribution between composite and SMU panel is not compromised.

In terms of practical application, the UC technique is simpler to apply than the DK method and it offers greater flexibility than the SV method.

On this basis, therefore, it can be concluded that of the three methods applied, the use of the UC method is most appropriate to this deposit.

References

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