Quantification of uncertainties in geological modeling of kimberlite pipes

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ABSTRACT

Future development of kimberlite mines is now focused on mining at deeper levels. The consequences are increased difficulty and costs of the mining operations. The decisions on investments require accurate resource evaluation and quantification of the risk. In the decision process the first step consists in choosing the number of boreholes sampling the pipe relative to the level of uncertainty that can be accepted.

The uncertainty in the volumes of rocks of different types can be assessed as soon as we can estimate these volumes from a given borehole layout and calculate the error by means of its variance for instance. In theory this can be achieved by using geostatistical techniques ("transitive theory"), but in practice the calculation of the error on volumes can be achieved in very particular cases, assuming some regular sampling patterns.

A different approach has been chosen based on simulations, giving maximum flexibility to reproduce the reality of the sampling procedure. The idea is to simulate the geometry of several "possible" pipes with their internal geology and estimate them by kriging. Several experimental errors are obtained on which risk analysis can be performed.

The aim is to find a feasible method to perform the simulations with reasonable chance to correctly represent the reality, without data on the pipe extension. The methodology applied is based on some hypotheses that can be assessed by the geologist as well as by the geostatistician. The procedure assumes a geological model based on current knowledge including appropriate geostatistical variation. What make these simulations specific and powerful in this case comes from the fact that the boundaries between the different rock-types can be simulated using a transformation of the real 3D space into a 2D space defined by means of polar coordinates. Ultimate transformations to account for smoother variations with morphological transformations allow the generation of realistic pipe geometries on which sampling pattern can be used to evaluate the estimation errors.

By estimating the simulated pipe geometries with different boreholes layouts a risk curve expressing the uncertainty in the volumes of the different rock-types versus the number of holes can be developed.

INTRODUCTION

South African diamond mining of kimberlites dates back over 100 years. During this time, several of the pipes have been depleted and the mines closed, but a number of mines have extensive resources at depths below their current infrastructural capacity. Exploitation of these resources will require substantial capital investment. Risk has become a major driving force in evaluation of these resources. Any delineation or drilling program has to ensure that the delineated resource will be within acceptable risk levels. Designing drilling campaigns has usually been a subjective process. Following recent investigations , a methodology to plan drilling programs to meet acceptable risk targets will be demonstrated.

This risk can be measured in different ways, all related to the uncertainty on the recoverable volumes that can be estimated from borehole data. The aim of the method is then to quantify the relationship between the number of boreholes planned and the confidence in the estimates of the annual production, roughly corresponding to a mining level.

All of the calculations and graphic output generated was made using the Isatis[1] geostatistical software. The illustrations come from an implementation on two different mines.

METHODOLOGY

The uncertainty or risk is nothing but a measure of the difference between estimation and reality. In geostatistics this corresponds to the concept of estimation error. On a basis of a variogram it is possible to calculate, in advance, the variance of this error, called the kriging variance. In this case the estimation variance on the volume (level by level) of different rock-types is the issue of concern. Such a problem of volume estimation can be solved by means of transitive geostatistics in the case of regular sampling. As it is not the case, calculating the estimation variance by applying ordinary kriging, which does not in theory require any data could be considered. However, this would imply that the volume on which the estimation is performed is fixed. As the volume is the object of the estimation, this implication is invalid.

The approach adopted was to calculate “experimental errors” by comparing the reality and an estimate of the reality based on borehole data. As the reality is unknown, a simulation is substituted for reality to produce a reasonable estimate of the error. The simulation must reproduce the spatial structure of correlation, i.e. the variogram, of the actual geology.

The difficulty is that there are no numeric data in the volumes under investigation, i.e., the lower extensions of the pipe. However a geological model with a good level of confidence is available. Whilst the geological model simplifies the irregularity of the boundaries, it is a suitable base case that can be used. In the simulation study, variations around the base case can be generated, using the geostatistical characteristics of the levels above the zone to be simulated. Geological mapping on the current and historical mining levels provide the geostatistical input to the simulation.

The geometry of the pipe and the internal geological geometries are represented by their boundaries. The simulation technique can quite easily handle such variables because of the relatively regular nature of kimberlite pipes. On a practical basis, it is geologically acceptable to consider that, at the different levels, a radius from a virtual center point defines the boundaries between the rock-types. In order to obtain a simulated block model in 3D space, it is necessary to perform the simulations in a 2D space with polar coordinates. In this new working space it is possible to express, and consequently simulate or estimate, the radius as a function of the azimuth and the level. After having performed the simulation/or the estimation of the radius the simulation reverts to the original 3D space, by assigning the rock-type code to all blocks with a given azimuth and level up to the simulated/estimated radius.

The different steps are:

- Simulation of the pipe/internal geometries:
  - Calculation of the radius of the boundaries from the geological model
  - Geostatistical simulations of residuals that are added to the Base Case model
  - Assignment of the rock-types for each block in the simulated models
- Sampling from pseudo boreholes and determination of the intercepts with the simulated boundaries

- Estimation of the radius of the boundaries using the borehole intercepts for each simulation.
- Statistical analysis of the results

GEOSTATISTICAL PROCEDURE

Simulation of the pipe geometries

Figure 1 is a simplified representation of the Base Case model that has been used for one of the mines. Three different rock-types with the most significant differences in terms of economic potential were retained from the original model.

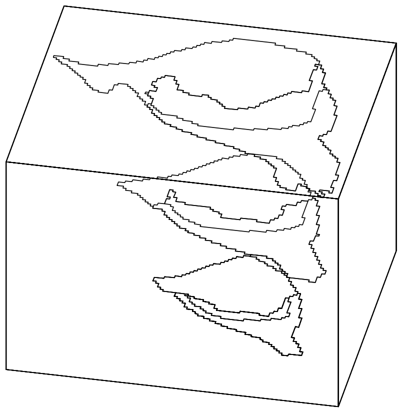

Figure 1. Geological model of three rock-types at 3 levels

The geometry of the pipe appears as a rather simple shape with dimensions increasing from bottom to top. This is coherent with current understanding of emplacement mechanisms of kimberlite pipes. (Field and al 1998[2]). For any level the shape can be described by means of a parametric function where the coordinates of the boundary depends on the azimuth and the radius from a center point. If the radii were constant, a cone with a circular base would result. By varying the radius with the azimuth and the level, any required shape can be obtained, the only condition being that a one to one relationship exists between radius and azimuth. By digitizing the boundary from the input geological model at one degree intervals a mathematical expression for the pipe boundary can be obtained. By superimposing a 3D block model each block can be described as belonging inside or outside the pipe. The vertical resolution of the block model should coincide with the resolution of the geological model, chosen as the height of the blast levels in this case.

As explained in the previous section, the uncertainty in the geometry will be characterized by adding some fluctuation around the geological model. The representation by a parametric function of (azimuth, radius, level) is perfectly suitable to that task. The radius is considered as a regionalized variable in the (azimuth, level) bi-dimensional space. As with all regionalized variables, this variable can then be processed by means of geostatistical simulation and kriging. It is appropriate to adopt a non-stationary viewpoint such that the geometry can be guided by the geological model. The radius of the pipe boundary is then decomposed into the sum of the radius from the geological model and a stationary 0-mean residual. The geostatistical characteristics derived from the informed blocks higher up in the body are used to characterize these residuals as there are no data available in the area to be simulated.

Detailed mapping of the pipe boundaries of a few production levels above the area of interest are available. Using these, it is possible to calculate the variogram of the radius as a function of the azimuth for each level. As expected these variograms show similarities, and in this case a model was fitted to the average variogram. The fitted variogram model is a stationary model with two structures one with a short range and another one with a long range. Beyond a “distance” of 100 degrees the increasing of the variogram reveals the non-stationarity. Making a non-stationary model with generalized covariances is a difficult task with these data. A more simplified approach was preferred, considering the short range structure as representing the random fluctuations of the residuals and relating the long range structure to the trend.

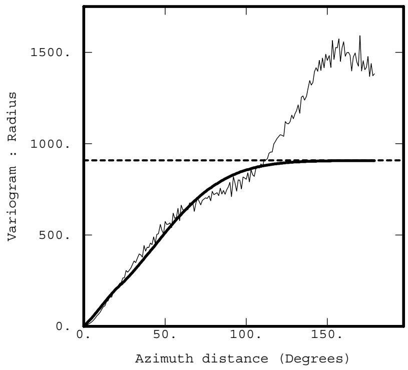

Figure 2. Average horizontal variogram of the radius of the pipe boundary.

An unusual aspect of the geostatistical analysis of this particular variable , is its "circular" aspect: i.e. the radius is defined modulus 2π, the maximum distance for considering the correlation should be 180 degrees and not 360. A variogram model suitable for this feature is not known. By using the variogram the focus is on the short distances, making the cyclic type of the variable of minor importance in the geostatistical modeling. At the simulation stage this feature cannot be ignored as it is expected that the simulation at azimuth 359 should be highly correlated to the simulation at azimuth 0. The variogram implies no correlation at the pseudo distance of 359 degrees. In order to overcome this difficulty the simulation was undertaken in two steps. Firstly a non-conditional simulation of the residuals from 0 to 180 degrees was performed. Secondly, the simulated residuals for 0 and 180 degrees were retained as input data for making conditional simulations from 181 to 359 degrees.

The simulation of the residuals is achieved by using the turning bands[3] method in the 2D space (azimuth-level). This requires the variogram in that space. In the absence of real data that can be used to determine the spatial structure along the vertical axis a empirical approach was adopted. Most of the vertical variations of the pipe geometry come from the trend and consequently the variations for levels in close vertical proximity of each other would be similar. By an iterative process vertical range was arbitrarily fixed to reproduce this property.

Figure 3 shows the outlines of the geological model and a simulated pipe boundary for one level.

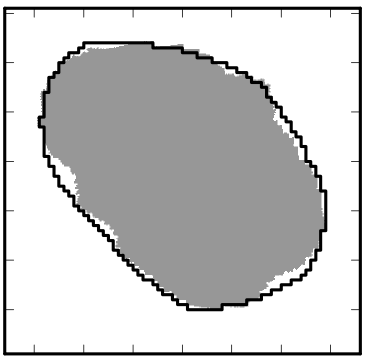

Figure 3. Simulated pipe boundary with the outlined boundary from the geological model.

The generation of the simulated geometries is obtained by means of stochastic simulation that produces characteristics that render these images unrealistic. In particular, the jagged outlines that lead to reversed slopes in vertical sections (above 90 degrees). I.e. pipe dimensions not consistent with an increase from bottom to top. Instead of introducing such constraints (regularity of the outline, admissible slopes) at the simulation stage, a post-processing of the simulated models aiming at regularizing the geometry according to shape criteria has been applied. These techniques originate from mathematical morphology methods. Figure 4 shows a section before and after such transformation. The resultant simulated models are considered as acceptable from a geological and statistical perspective, since they produce different images of the same geological phenomenon with a reasonable variability.

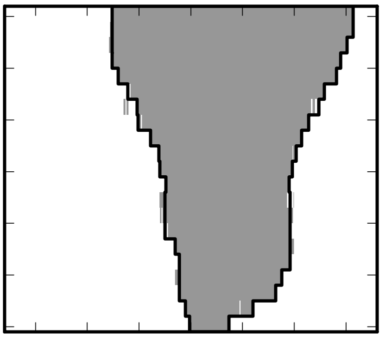

Figure 4. Vertical simulation of the pipe before and after regularization of the outlines.

Once the pipe geometry has been simulated, the simulation of the internal geology has to be attempted. This is beyond the scope of this paper, suffice to say that the principle of the method for simulating the pipe perimeter cannot be simply extended to simulating any geometry. The real geology must be simplified in any case, but the ability to adapt the approach used for the pipe boundary to internal geology depends on the general shape of the internal boundary. In the example shown in Figure 1, the internal waste at the center of the pipe "the croissant" cannot be considered as an "internal pipe" that could be patched inside the kimberlite pipe as there is no center point from which one can draw a radius that has only one intersect with the boundary of that geology. The method to simulate the internal geology was adapted from the pipe simulation by intersecting two "pseudo pipes" generated from the external and the internal radius of the "croissant". It is certainly not suggested that any geometry can be simulated using this method, however some flexibility exists as long as the approximation does not contradict the major features of the geology.

Estimation from Boreholes

After multiple simulations of the pipe and its internal geology a number of times, different ways of sampling can be considered and the estimation of the volumes for each rock type using these samples can be undertaken. If the borehole layout corresponding to a given sampling scenario is fixed, the same estimation procedure can be applied to each simulation. The simulation plays the role of the unknown reality, so the difference between the simulated and the estimated volume represents the estimation error. By repeating the process on every simulation many outcomes of the estimation error are obtained on which statistics can be calculated.

The estimation method is based on kriging by following a similar approach to the simulation step., i.e. the radius in the 2D (azimuth-level) space is estimated. These data are derived from the coordinates of the intercepts of the boreholes with the simulated pipe. The difference from the simulation step is that the geological model is ignored and the radius is estimated directly, rather than using the model plus residual. By using ordinary kriging with moving neighborhood, the variogram model of the type shown in Figure 2 can be used instead of a really non-stationary model, which is more difficult to infer.

The borehole layouts have been designed taking cognizance of technical constraints. For both mines studied to date two different types of layouts have been considered (Figure 5).

For the first mine (on the left) the boreholes are drilled horizontally in fans from two opposite sides of the pipe. The sampling scenarios deal with the number of holes per level and the number of levels. It was also considered that some boreholes might miss the pipe (and internal geology) boundaries, as shown in Figure 5. A random process generates subsets of the boreholes.

For the second mine (on the right) inclined boreholes are drilled from a level above. The information provided by vertical holes has been ignored as they are essentially useless in providing any data on the boundaries between the different rock types.

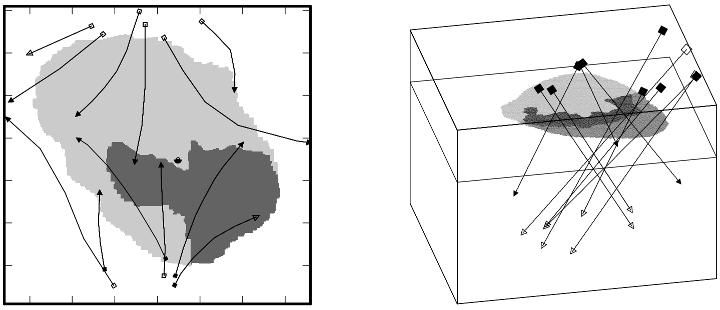

Figure 5. Boreholes layouts for estimating the rock volumes.

Kriging tends to over-smooth the actual variability, so it is expected that the kriged outlines of the pipe should look much more regular than the simulated (real) ones. The best interpolation between the data is to draw an arc of a circle for joining two data points as it is illustrated in figure 6. By adding more information the actual boundaries are better approximated and the smoothing effect of kriging decreases.

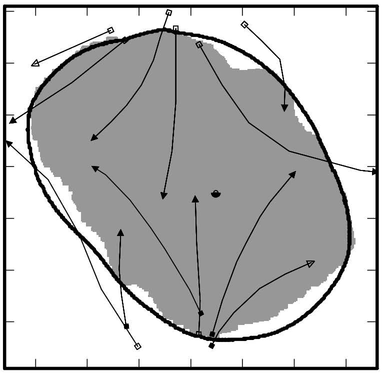

Figure 6. Comparison of simulated pipe and the kriged outlines

An attempt was made to maximize the information from the boreholes by using indirect information as to whether or not the informed blocks are inside or outside the pipe. This information can be used in the kriging system by replacing the radius value by a minimum or a maximum bound. This particular type of kriging mixing hard and soft data is known as kriging with inequalities. The results were unconvincing as no improvement was noted until quite large numbers of hard data are available. Unless the sample density reaches a certain level the introduction of these inequalities actually appeared to introduce additional uncertainty. The standard kriging technique was therefore retained.

Analysis of the results

As this is still a work in progress, all results are very preliminary and subject to re-interpretation. In particular the following results are based on ten simulations of the pipe, but it appears that it already provides reliable information on the relationship between the number of boreholes and the uncertainty in the estimation of volumes. Only results on the total pipe volume for the first mine where the boreholes are drilled horizontally are shown hereafter, but the same analysis can be done on the volumes for each rock type. Initial analysis of the data was to compare the volumes per level (roughly corresponding to the annual production) from the simulations to the volume per level based on the estimates. This is shown below in figure 7. The average volumes from the 10 simulations are plotted for 4 different sampling scenarios, reducing the number of holes from 104 to 42 holes, and for the original simulations.

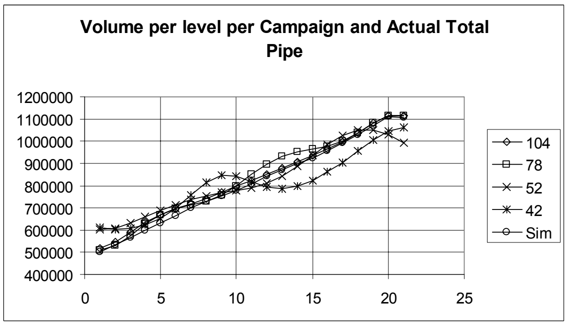

Figure 7 Volume (m^3) vs. level (bottom level is #1)

It can be seen how the how the departure of the estimated figures from the simulation tends to increase when reducing the number of holes. The error and the absolute error (difference between the estimate and the simulation were calculated and the histogram of the absolute error is given in Figure 8 for the scenarios with 104, 78 and 52 boreholes. The distribution of errors is clearly more spread out for a lower number of boreholes.

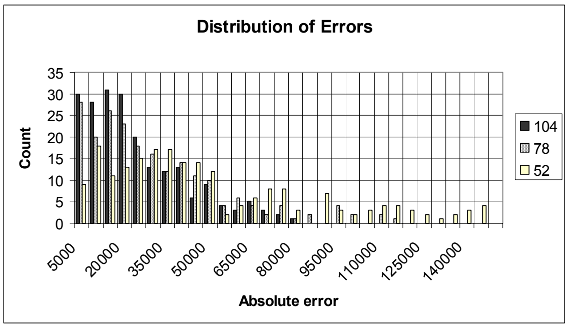

Figure 8. Distribution of the absolute errors per level per estimate.

One difficulty in the analysis arises from the fact that the support of the estimation is changing with the level (the pipe dimensions decrease with depth). Working in relative values does not solve the problem, so it is important to compare the different sampling scenarios level by level. From the distribution of the error two statistical representations were retained, which whilst being synthetic, allow a more detailed analysis. The most powerfull statistics consist of the average of the error (on 10 simulations), called MBE (Mean Biased Error) and of the average of the absolute error, called MAE (Mean Absolute Error). MBE evaluates the bias of the estimation while MAE evaluates the spread of the error. MAE has been preferred to the standard deviation of the error because it is less sensitive to outliers.

In figure 9, representing the MBE, it can be observed that irrespective of the number of boreholes a residual bias remains. This is probably a function of the fixed borehole layout which may miss some feature of the geometry whatever the simulation. This would explain the occasionsal instances where the estimate improves with fewer boreholes. by chance.

Figure 10 represents the MAE. Two conclusions can be drawn. Firstly the error shows a clear tendency to decrease with the number of boreholes and secondly, the variability of the error from one level to another one is directly linked to the number of boreholes. With the maximum number of boreholes the estimates of the volumes are of approximately the same quality for all levels

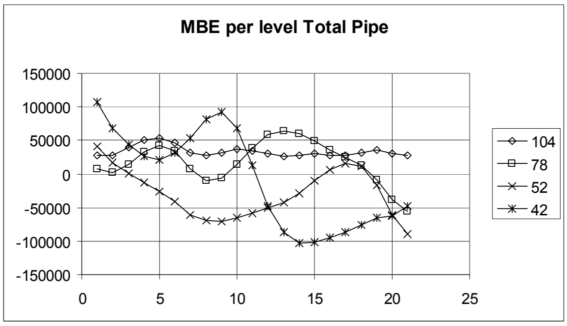

Figure 9. The mean biased error per level

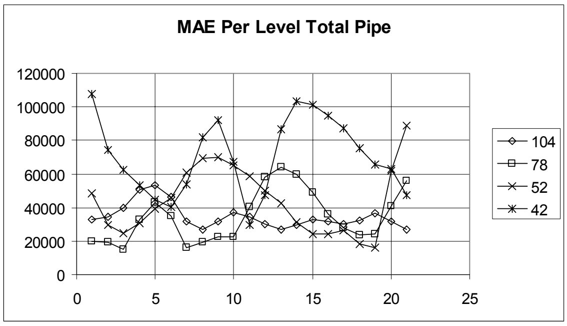

Figure 10. Mean absolute error per level.

Having checked the detailed results level by level, the mean absolute error for all levels combined can be considered as a function of the number of boreholes (see figure 11). Therefore, each point on the graph is an average of 210 (21 levels by 10 simulations) values of errors. Although these statistics mix different support, the result is rather spectacular, showing that the uncertainty on the volumes drops drastically when the number of boreholes increases from about 50 to 80. It is argued that the increase in error with 104 boreholes can be attributed to statistical fluctuations. It proves nevertheless the existence of a critical number of boreholes that is lower than the maximum that was envisaged in this study. To get a more precise determination of the "optimal" sampling layout would require the examination of more scenarios and probably also more simulations.

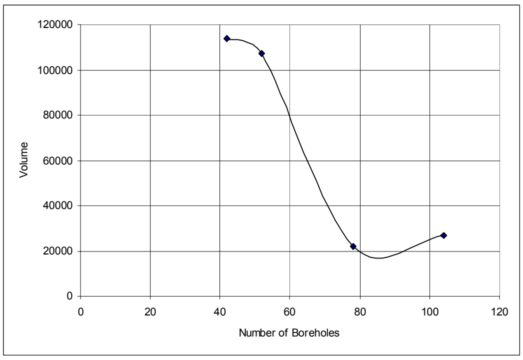

Figure 11. Global mean absolute error of the volumes per level.

From this point it is possible to extend the scope of the analysis by looking at the relationship between the number of boreholes (and the cost per meter drilled) and the level of acceptable risk. This can be achieved by using simple models based on the assumption of a gaussian distribution of errors that should make it possible to relate the standard deviation of the error (or its substitute MAE) to a probability and a confidence interval.

CONCLUSIONS

This paper has demonstrated the practical application of geostatistical non-conditional simulations for making decisions on the optimal sampling strategy of kimberlite pipes. It is expected that the uncertainty in estimating the resources decreases when adding more data. With this geostatistical approach an important step is made towards quantifying the relationship. By choosing a given level of risk (or confidence) the decision on the borehole layout is no longer arbitrary. In addition, the simulations provide visual support to the mining engineers when considering technical problems.

Whilst the method is not universally applicable, the approach was successfully applied to two mines within the De Beers group. This has been achieved by making minor changes to the methodology to accommodate the different geological morphologies. To be applicable, it must be possible to represent the boundaries of the pipe and internal geology by a radius from a center, with a unique value for each direction of the space. The main obstacle arises from the "circular" nature of that variable. This original solution has been fully implemented using a batch procedure based on ISATIS software.

In conclusion, an effective analysis and decision tool has been developed with wide applications. Its use is dependant upon

- Defining the base models and designing various sampling campaigns,
- Investigation of alternative solutions to the parameters of the model
- Provision of an effective framework for the organizing and structuring of the results as the procedure can be run many times, generating huge volumes of information.

Whilst all current work has been focused on modeling an area where no data are available, extensions to the system are envisaged to incorporate information from boreholes as they are being drilled to condition the simulations. This would be a powerful tool to optimize sampling.

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