The information effect and estimating recoverable reserves

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ABSTRACT

Recoverable reserves estimation, either globally or locally, has become a standard geostatistical application in the mining industry. Early in the analysis of the recoverable reserves problem, in the 1970's, both the support effect and the information effect were identified as playing a potentially important role in the final result. To date however only the support effect has been implemented to any significant extent. The information effect refers to the fact that, even during production, the real mining block grades are not known. Only an estimated value of them, based on production samples, is known and pay blocks are defined according to whether this value, not the real grade, is above the economic cut-off. So some pay blocks will be misclassified as waste and visa versa. This information effect, that quantifies the amount and the effect of the misclassification on the recoverable reserves must be taken into account to obtain a more realistic recoverable reserves estimate.

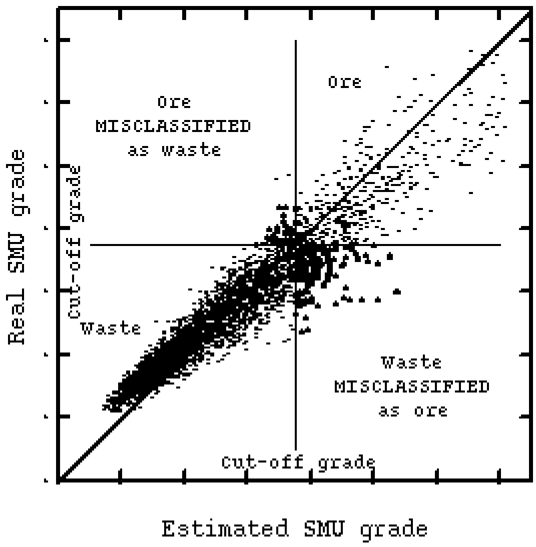
In this paper we show how the estimation of the global recoverable reserves and of the local reserves by uniform conditioning can be adapted to take the information effect into account. The multi-gaussian framework means that the distribution of both the real and estimated block grades can be modelled and incorporated into the estimation. The proposed technique will be applied to a case study based on a large Australian open-pit gold mine.

INTRODUCTION

In the geostatistical sense, recoverable reserves means the ore tons *T* and metal quantity *Q* contained in those tons for a given mining block or selective mining unit (SMU). The average recoverable grade *M* equals the metal divided by the tonnage: *M* = *Q*/*T*. Instead of dealing in absolute tons we consider the tonnage as the number of SMU's whose grade Z_v is above the cut-off grade z_c. The metal is calculated as this tonnage multiplied by the SMU grade. This type of formalism is used to express the two variables in terms of indicator variables :

$$T_v = I_{Z_v \geq z_c} = \begin{cases} 1 & \text{if } Z_v \geq z_c \\ 0 & \text{otherwise} \end{cases} \quad \text{and} \quad Q_v = Z_v I_{Z_v \geq z_c} = \begin{cases} Z_v & \text{if } Z_v \geq z_c \\ 0 & \text{otherwise} \end{cases} \qquad (1)$$

To calculate the absolute tonnage and metal, T_v and Q_v are multiplied by the SMU volume and the density of the ore. However, the expressions in (1) assume that at the time of mining the real SMU grade Z_v is known. This is not true. During production only an estimate Z_v^* of this grade is known and calculated from production samples. The SMU's sent to the mill as ore are those whose estimated grade, not necessarily real grade, is above the economic cut-off. So some real pay blocks will be misclassified as waste and some waste misclassified as ore. The amount of misclassification is due to the so-called information effect, selecting SMU's on the estimated grade and not the real one. Figure 1 presents a schematic example of the information effect where real SMU grades are plotted on the vertical axis against the estimated SMU grades on the horizontal. The estimate shown here is conditionally biased, leading to a dangerous overclassification of ore blocks for a cut-off grade above the global mean grade. For a non conditionally biased estimated the two types of misclassification will at least annihilate each other in terms of tonnage but not in terms of recovered grade.



So the reserves in (1) above are in fact "ideal" values. What should really be considered are the "actual" reserves that take the information effect into account:

$$T_v^* = I_{Z_v^* \geq z_c} \quad \text{and} \quad Q_v^* = Z_v I_{Z_v^* \geq z_c} \quad (2)$$

which can only be calculated if we model the distribution of both the real SMU grade and its estimator, as well as their joint and conditional distributions.

Although the information effect problem has long been recognised in geostatistics, the few theoretical developments in this direction have still not become part of mainstream industrial applications. Theoretical expressions for the "actual" global recoverable reserves have been developed (Matheron (1976), Lantuéjoul (1990)) within the multi-Gaussian framework. Matheron (1984) provided a simple lognormal example to highlight the potential importance of the information effect on the resulting grade tonnage curves. There is a proposed modification (e.g. Remacre (1989)) to the calculation of the local recoverable reserves by uniform conditioning by panel grade (UC) that takes into account the fact that the real panel grade is approximated by its kriged estimate. Very recently, working on UC, Assibey-Bonsu & Krige (1999) suggest correcting the smoother estimated SMU and panel grades to reproduce the real variability in these estimates before applying the cut-off. In this paper, we will adapt the global approach and combine it with the proposed local modification to produce a recoverable reserves estimate by UC that consistently incorporates the two different sources of information effect. Let us begin by recapitulating the global reserves case.

GLOBAL RECOVERABLE RESERVES

The "Ideal" Reserves

Globally we are not interested in the number of SMU's above cut-off but the proportion of the orebody, in terms of SMU's, that can be considered as ore; in other words, the probability that an SMU chosen at random is above cut-off. Under an ergodic assumption, this proportion is equal to the mean of the indicator function:

$$T_v(z_c) = \mathrm{E}\{\mathrm{I}_{Z_v \geq z_c}\} = \mathrm{Prob}\{Z_v \geq z_c\} = 1 - \mathrm{Prob}\{Y_v < y_c\} \quad (3)$$

where, according to the discrete Gaussian change of support model:

$Z_v = \Phi_r(Y_v) = \sum_{n=0}^{\infty} \phi_n r^n H_n(Y_v)$ such that r is the correlation coefficient of the supposedly standard bi-Gaussian pair $\{Y_{\underline{x}}, Y_v\}$, $\underline{x}$ is a point chosen at random in v and y_c the Gaussian equivalent SMU cut-off $y_c = \Phi_r^{-1}(z_c)$. H_n denotes the nth standardised Hermite polynomial. The metal quantity can then be written as:

$$Q_v(z_c) = \mathrm{E}\{Z_v \mathrm{I}_{Z_v \geq z_c}\} = \mathrm{E}\{\Phi_r(Y_v)\mathrm{I}_{Y_v \geq y_c}\} = \int_{y_c}^{\infty} \Phi_r(y) g(y) dy \quad (4)$$

Incorporating the Information Effect - the "Actual" Reserves

The "actual" global reserves are therefore defined as:

$$T_v^*(z_c) = \mathrm{Prob}\{Z_v^* \geq z_c\} = \mathrm{Prob}\{Y_v^* \geq y_c\} \quad \text{and}$$

$$Q_v^*(z_c) = \mathrm{E}\{Z_v \mathrm{I}_{Z_v^* \geq z_c}\} = \mathrm{E}_{Z_v^*}\left[\mathrm{E}\{Z_v \mathrm{I}_{Z_v^* \geq z_c} | Z_v^*\}\right] = \mathrm{E}\left[\mathrm{E}\{Z_v | Z_v^*\}\mathrm{I}_{Z_v^* \geq z_c}\right]$$

which can be calculated once the distribution of Z_v^* and $\mathrm{E}\{Z_v | Z_v^*\}$ have been modelled. For a conditionally unbiased estimator $\mathrm{E}\{Z_v | Z_v^*\} = Z_v^*$ and the reserves are defined by the distribution of Z_v^* alone.

We then assume that Z_v^* can be written as: $Z_v^* = \sum_{i=1}^{m} \lambda_i Z(x_i)$ such that $\sum_{i=1}^{m} \lambda_i = 1$ where there are m production samples $Z(x_i)$ used to estimate the mining block v and λ_i is the (kriging) weight associated with the ith sample such that $\lambda_i \geq 0 \ \forall i$. Let I designate the random sample number such that Prob{$I = i$} = λ_i. Then $Z(x_I)$ has the same distribution as the samples and:

$$\mathrm{E}\{Z(x_I) | Z_v^*\} = \sum_{i=1}^{n} \lambda_i \mathrm{E}\{Z(x_i) | Z_v^*\} = Z_v^* \quad (6)$$

which is analogous to Cartier's relationship. So if we write:

$Z(x_I) = \Phi(Y(x_I)) = \sum_{n=0}^{\infty} \phi_n H_n(Y(x_I))$ and assume that $Y(x_I)$ and Y_v^* form a standard bi-Gaussian pair then from (6):

$$Z_v^* = \mathrm{E}\left\{ \sum_{n=0}^{\infty} \phi_n H_n(Y(x_I)) \middle| Y_v^* \right\} = \sum_{n=0}^{\infty} \phi_n s^n H_n(Y_v^*) = \Phi_s(Y_v^*) \quad (7)$$

where the coefficient s = Corr$(Y(x_I), Y_v^*)$ is calculated by inverting the following variance identity:

$$\sum_{i=1}^{m} \sum_{j=1}^{m} \lambda_i \lambda_j C(x_i - x_j) = \mathrm{Var}(Z_v^*) = \sum_{n=1}^{\infty} \phi_n^2 s^{2n} \quad (8)$$

where $C(x_i - x_j) = \mathrm{Cov}(Z(x_i), Z(x_j))$ is the sample grade covariance function.

The distribution of $Z_v | Z_v^*$ is modelled in a very similar way. Let us assign $\rho = \mathrm{Corr}(Y_v^*, Y_v)$ as the correlation coefficient of the supposedly bi-Gaussian pair (Y_v^*, Y_v). We then find:

$$\mathrm{E}\{Z_v | Z_v^*\} = \mathrm{E}\{\Phi_r(Y_v) | Y_v^*\} = \sum_{n=0}^{\infty} \phi_n r^n \rho^n H_n(Y_v^*) \quad (9)$$

where ρ is calculated from the following covariance identity:

$$\sum_{i=1}^{m} \lambda_i \frac{1}{\mathrm{Meas}|v|} \int_{x \in v} C(x_i - x) dx = \mathrm{Cov}(Z_v^*, Z_v) = \sum_{n=1}^{\infty} \phi_n^2 r^n s^n \rho^n \quad (10)$$

From (7) and (9), we obtain the calculable expressions for the "actual" global reserves:

$$T_v^{\ddagger}(z_c) = \mathrm{Prob}\{Y_v^* \geq y_c^*\} = 1 - G(y_c^*) \quad \text{and}$$

$$Q_v^{\ddagger}(z_c) = \mathrm{E}\{\mathrm{E}[Z_v | Z_v^*] \mathrm{I}_{Z_v' \geq z_c}\} = \mathrm{E}\{\Phi_{r\rho}(Y_v^*) \mathrm{I}_{Y_v' \geq y_c'}\} = \int_{y_c'}^{\infty} \Phi_{r\rho}(y) g(y) dy \quad (11)$$

where $y_c^* = \Phi_s^{-1}(z_c)$ is the Gaussian equivalent cut-off grade applied to the distribution of the estimated SMU grades, and where g is the standard Gaussian probability density function.

LOCAL RECOVERABLE RESERVES - UNIFORM CONDITIONING

The "Ideal" Reserves

Using a similar approach, the information effect can be taken into account when calculating local recoverable reserves using UC by panel grade. According to this technique we suppose there are many SMU's per panel and for each panel we estimate the proportion of SMU's above cut-off and the corresponding metal quantity. The "ideal" probability that an SMU chosen at random within a panel is above cut-off, given the known average panel grade Z_V, is calculated as:

$$T_v(z_c) = \mathrm{E}\{\mathrm{I}_{Z_{\underline{v}} \geq z_c} | Z_V\} = \Pr\{Z_{\underline{v}} \geq z_c | Z_V\} = 1 - \mathrm{Prob}\{(Y_{\underline{v}} | Y_V) < y_c\} \quad \text{and}$$

$$Q_v(z_c) = \mathrm{E}\{Z_{\underline{v}} \mathrm{I}_{Z_{\underline{v}} \geq z_c} | Z_V\} = \mathrm{E}\{\Phi_r(Y_{\underline{v}}) \mathrm{I}_{Y_{\underline{v}} \geq y_c} | Y_V\}$$
$$= \mathrm{E}\left\{\sum_{n=0}^{\infty} q_n H_n(Y_{\underline{v}}) | Y_V\right\} = \sum_{n=0}^{\infty} q_n R^n H_n(Y_V) \qquad (12)$$

where Y_V is the Gaussian equivalent panel grade. We suppose that the function $\Phi_r(Y_v)\mathrm{I}_{Y_v \geq y_c}$ can be written as a development in terms of H_n with its specific coefficient q_n. *R* is the change of support coefficient from SMU's to panels, that is the correlation coefficient of the supposedly bi-Gaussian pair $(Y_{\underline{v}}, Y_V)$: $R = \mathrm{Corr}\{Y_{\underline{v}}, Y_V\}$. $\underline{v}$ designates a SMU chosen at random within the panel *V*.

Incorporating the Information Effect - the "Actual" Reserves

There are two sources of information effect in (12). Firstly, Z_V is not known but estimated Z_V^* from exploration samples, and secondly, as in the global case, the SMU's will ultimately be selected as pay blocks on a future estimated grade, not the true one. The "actual" reserves are therefore:

$$T_v^*(z_c) = \mathrm{E}\{\mathrm{I}_{Z_{\underline{v}}^* \geq z_c} | Z_V^*\} = \mathrm{Prob}\{Z_{\underline{v}}^* \geq z_c | Z_V^*\} = 1 - \mathrm{Prob}\{(Y_{\underline{v}}^* | Y_V^*) < y_c^*\} \quad \text{and}$$

$$Q_v^*(z_c) = \mathrm{E}\{Z_{\underline{v}} \mathrm{I}_{Z_{\underline{v}}' \geq z_c} | Z_V^*\} = \mathrm{E}\{\Phi_r(Y_{\underline{v}}) \mathrm{I}_{Y_{\underline{v}}' \geq y_c'} | Y_V^*\}$$
$$= \mathrm{E}_{Y_{\underline{v}}'}\left[\mathrm{E}\{\Phi_r(Y_{\underline{v}}) \mathrm{I}_{Y_{\underline{v}}' \geq y_c'} | Y_V^* Y_{\underline{v}}^*\}\right] = \mathrm{E}_{Y_{\underline{v}}'}\left[\mathrm{E}\{\Phi_r(Y_{\underline{v}}) \mathrm{I}_{Y_{\underline{v}}' \geq y_c'} | Y_{\underline{v}}^*\} | Y_V^*\right]$$
$$= \mathrm{E}\{\Phi_{r\rho}(Y_{\underline{v}}^*) \mathrm{I}_{Y_{\underline{v}}' \geq y_c'} | Y_V^*\} = \mathrm{E}\left\{\sum_{n=0}^{\infty} q_n^* H_n(Y_{\underline{v}}^*) | Y_V^*\right\} \qquad (13)$$

where $Y_V^* = \Phi_p^{-1}(Z_V^*)$ is the Gaussian equivalent estimated panel grade. Again the function $\Phi_{r\rho}(Y_v^*)\mathrm{I}_{Y_v' \geq y_c'}$ is written in terms of H_n with coefficients q_n^*. To evaluate the quantities in (13) we must model the distribution of $Y_{\underline{v}}^* | Y_V^*$.

Firstly we model the distribution of Z_V^* like that of Z_v^* in (7), based on similar hypotheses but relating to the estimated panel grade: $Z_V^* = \sum_{i=1}^{m'} \mu_i Z_i$ such that $\sum_{i=1}^{m'} \mu_i = 1$ where there are *m?* exploration samples Z_i used to estimate the panel *V* and $\mu_i \geq 0 \ \forall i$. We find:

$$Z_V^* = \mathrm{E}\left\{\sum_{n=0}^{\infty} \phi_n H_n\left(Y_{x_i}\right) \middle| Y_V^*\right\} = \sum_{n=0}^{\infty} \phi_n p^n H_n\left(Y_V^*\right) = \Phi_p\left(Y_V^*\right) \qquad (14)$$

where the coefficient $p = \mathrm{Corr}\left(Y_{x_i}, Y_V^*\right)$ is calculated by inverting the following identity:

$$\sum_{i=1}^{m'} \sum_{j=1}^{m'} \mu_i \mu_j C(x_i - x_j) = \mathrm{Var}\left(Z_V^*\right) = \sum_{n=1}^{\infty} \phi_n^2 p^{2n} \qquad (15)$$

We then suppose that the couple $\left(Y_{\underline{v}}^* Y_V^*\right)$ is bi-Gaussian with correlation coefficient *t*. Given that $\underline{v}$ is a randomly chosen SMU in the panel *V*, we use the identity:

$$\sum_{i=1}^{m'} \mu_i \frac{1}{\mathrm{Meas}|V|} \int_{x \in V} C(x_i - x)dx = \mathrm{Cov}\left(Z_{\underline{v}}^* Z_V^*\right) = \sum_{n=1}^{\infty} \phi_n^2 s^n p^n t^n \qquad (16)$$

to calculate *t* numerically, given that *s* and *p* are known. So given the conditional distribution of $Y_{\underline{v}}^* \middle| Y_V^* \sim N\left(tY_V^*, 1 - t^2\right)$, we can evaluate the "actual" tonnage and metal quantity for the panel *V*:

$$T_v^*(z_c) = 1 - \mathrm{Prob}\left\{Y_{\underline{v}}^* \middle| Y_V^* < y_c^*\right\} = 1 - G\left\{\frac{y_c^* - tY_V^*}{\sqrt{1 - t^2}}\right\} \quad \text{and}$$

$$Q_v^*(z_c) = \mathrm{E}\left\{\sum q_n^* H_n(Y_{\underline{v}}^*) \middle| Y_V^*\right\} = \sum q_n^* t^n H_n\left(Y_V^*\right) \qquad (17)$$

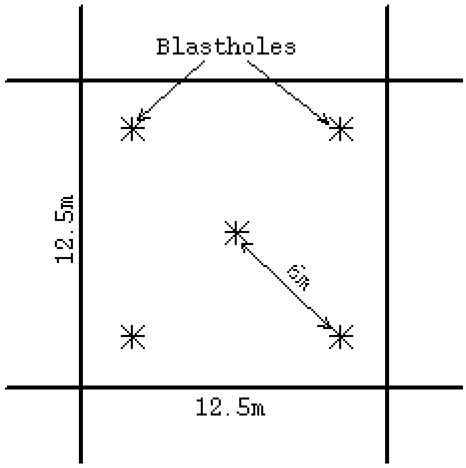
where $z_c = \Phi_s(y_c^*)$. The general form of the equations in (17) is very similar to that of the "ideal" case in (12). Only relatively minor changes need be made to the latter to obtain a local reserves estimation that takes the information effect into account.

PRACTICAL APPLICATION

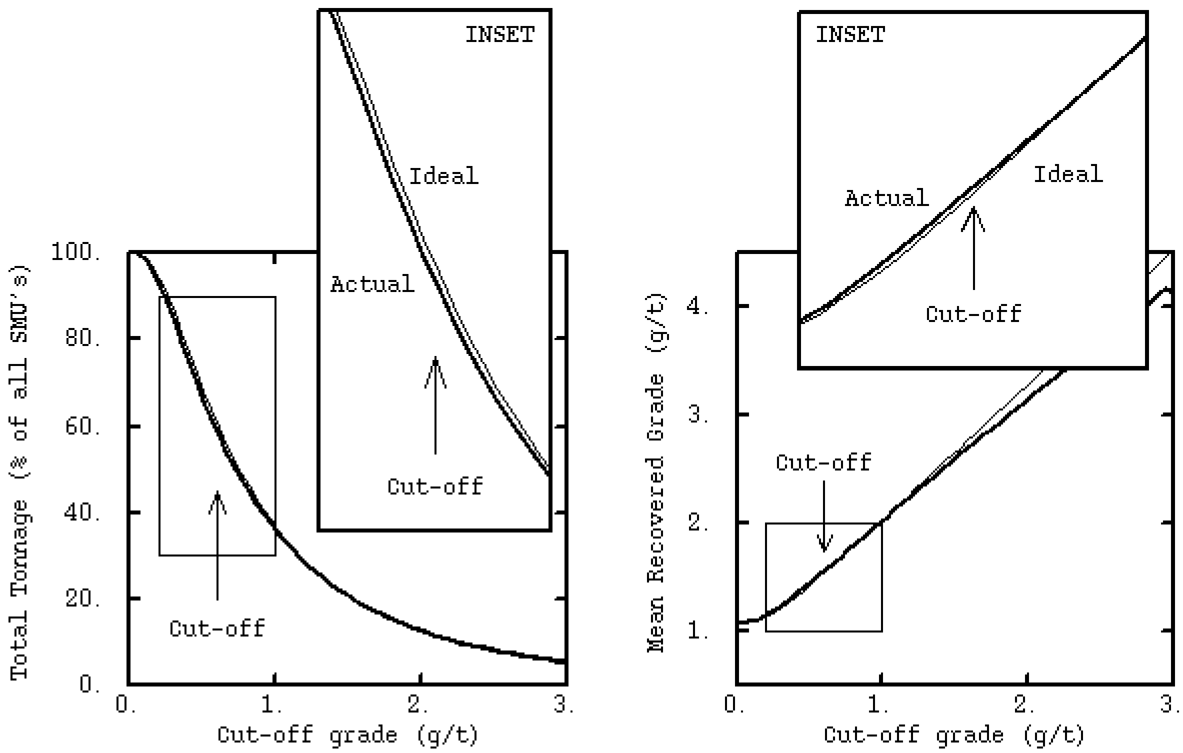
Our case study is based on the specifications of a large Australian disseminated gold (Au) deposit. All calculations were made using the Isatis software package. For confidentiality reasons, all values have been multiplied by a constant. The 3m Au composite grade histogram is positively skewed with a mean of 1.08g/t and a variance of 2.15 $(g/t)^2$. The Au variogram model is composed of a nugget effect (about 30% of the sill), an isotropic short range (60m) spherical structure (about 50% of the sill) and an anisotropic long range (from 120 to 270m) spherical structure. The main direction of anisotropy follows the orientation of the mineralization.

Global Recoverable Reserves: Ideal versus Actual

The global reserves are based on the distribution of SMU grades only. The proposed SMU size is 12.5 x 12.5 x 9m with a blasthole pattern based on an off-set 6 x 6m grid, as seen in Figure 2.



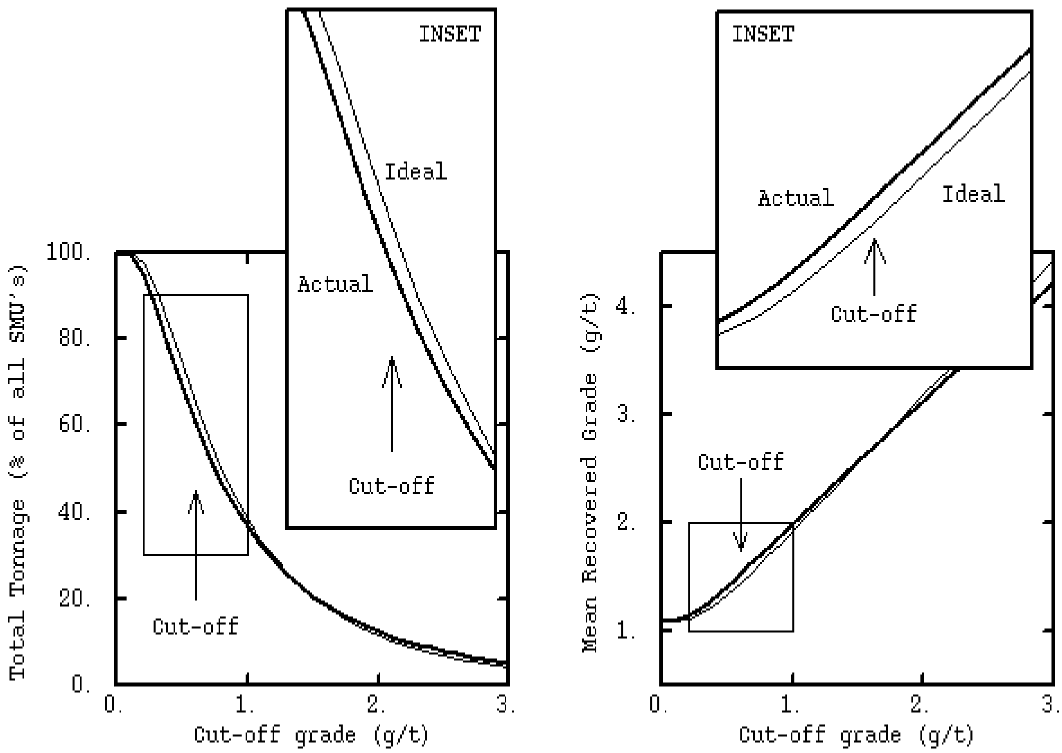
Knowing the Au composite grade variogram and the SMU size, Krige's relationship is used to calculate the relatively important "ideal" change of support r = 0.825 due to the moderately high nugget component. Given the blasthole pattern in Figure (2), we invert (8) to calculate s = 0.848, which is used to obtain the Gaussian equivalent cut-off grades y_c^*. Finally inverting (10) leads to the value r = 0.974. These values are inputted into (11) to produce global the grade-tonnage curves seen in Figure 3. The "ideal" reserves are shown as a thin line and the "actual" reserves as a thick line.



For illustrative purposes, the inset for each curve highlights the results obtained around the expected economic cut-off grade of the mine, 0.6g/t. For cut-off grades less than the global mean (of 1.08g/t) taking the information effect into account means actually recovering slightly less tons at a slightly higher grade. As the cut-off grade increases this trend is reversed. The number of tons actually recovered increases while the recovered grade is significantly lower than what would ideally be expected. This corresponds to the situation seen in Figure 1 where many more real waste blocks are misclassified as ore than visa versa. The difference between the "ideal" and "actual" results is because the SMU estimate based on only the 5 blasthole samples is conditionally biased, the theoretical slope of the regression $Z_v|Z_v^*$ being less than 0.88. On the other hand, for cut-offs lower than the global mean grade the maority of pay blocks are in fact underestimated and misclassified as waste. This leads to a somewhat surprisingly higher recovered grade but at the expense of less recovered tons.

Local Recoverable Reserves: Ideal versus Actual

Local reserves by UC are based on panel grade estimates. Because the exploration drilling grid is roughly 50 x 50m, the panel size was chosen as 50 x50 x 9m and panels were centred on the drilling grid. The quality of the panel estimates is quite good, the theoretical slope of the regression $Z_V|Z_V^*$ being typically greater than 0.95. The real panel grades have a theoretical variance of Var(Z_V) = 0.68 but this is not matched by the variability of the estimated panel grades: Var(Z_V^*) = 0.55 because of the smoothing properties of kriging. This difference is accounted for in (14) when calculating the change of support and information coefficient *p* = 0.607. Then knowing *s* and *p*, the relationship (16) is used to calculate the coefficient *t* = 0.661 that takes both the variability of the panel estimator and the SMU estimator into account. The UC results are presented globally in Figure 4 for direct comparison with the global estimation (cf. Figure 3).



While the differences between the "ideal" and "actual" results are more important than those seen in the global estimate, the trends are very similar. For low cut-offs, like the 0.6g/t of the mine, we actually recover less tons at a higher grade than that predicted had the information effect been ignored. For cut-off's greater than the global mean, ignoring the information effect means that the tonnage will be underestimated while the recovered grade will be overestimated, with the potential financial implications that may follow. Closer analysis showed that the most of the impact of the information effect is at the panel level - correcting for the reduction in variability between estimated and real panel grades. This is then compounded by the difference between real and estimated SMU grades for which the impact can be seen in Figure 3.

CONCLUSION

Incorporating the information effect leads to a more realistic recoverable reserves estimate. Moreover, this should become a mainstream industry standard given that only relatively minor modifications are required to existing estimation techniques to account for the information effect. The case study results show that the difference between "ideal" and "actual" global reserves depends on the dispersion and orientation of the scatter diagram between the real and estimated SMU grades, as well as the position of the economic cut-off grade relative to the global mean grade. On the other hand, most of the difference in the UC local reserves come from the use of estimated panel grades, instead of the real ones, when conditioning the result locally. Because of these different factors, it is not possible to generalise on the extent of the financial consequences that ignoring the information effect will have on the grade-tonnage curves of a particular mining project. This can only be evaluated on a case by case basis according to the specific characteristics of the project, so as to provide the most appropriate recoverable reserves estimate on which to base feasibility studies.

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